Engineering students approaching the mathematics textbook as a potential learning tool – opportunities and constraints

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Preface

This dissertation is a result of the research cooperation between the University of Agder and Narvik University College.

I am grateful that this study allowed me to deepen my knowledge within the area of learning and teaching mathematics at tertiary level and develop my research competences.

To my supervisors, Professor Barbro Grevholm from the University of Agder and Associate Professor Ragnhild Johanne Rensaa from Narvik University College, I want to express the deepest gratitude for sharing with me their knowledge and competence. Your guidance, support and challenging comments have been most valuable to me**.** I also want to thank very much my external supervisor Professor Lars-Erik Persson at the Luleå University of Technology.

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Mira Randahl
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Abstract

It is usually assumed that the students at tertiary level work intensively and individually with the new mathematical concepts (Wood, 2001). In this context the mathematics textbook might be an important learning tool. This thesis addresses the issue of what factors might influence the role of the mathematics textbook as a learning tool. The study is situated in the context of the basic mathematics course taken by first-year engineering students. A brief pilot study indicated that a majority of the students preferred using lecture notes rather than the textbook although in the beginning of the semester they perceived the textbook as being important when learning mathematics. This was the starting point of this research that aims to identify and explore the factors that might influence the role of the textbook proposed to first-year engineering students. The textbook is conceptualized as a cognitive learning tool embedded in the educational setting offering the basic mathematics course. The study was conducted when the students worked with the derivative concept. The process of students' approaching the textbook is viewed from an epistemological, a cognitive and a didactical perspective. The study is an exploratory case study, and a qualitative research strategy within an interpretative paradigm was chosen. Data was gathered from a number of vantage points: students' responses to a questionnaire, observations of lectures and task solving sessions, interviews with the teacher and the students, and informal talks with the teacher and the students. Additionally a questionnaire was sent to authors of the most used calculus textbooks with the aim of exploring their vision concerning their mathematics texts. The results of the main study confirm the observed phenomena from the pilot study. Students perceived the textbook as difficult and the lecture notes were preferred when working with the derivative concept. The textbook was used by the students mainly to read the examples and figure out possible procedures when working with the tasks. The findings of the study reveal possible opportunities and constraining factors of epistemological, cognitive and didactical nature. Students' poor previous knowledge, their approach to learning mathematics, the cognitive demands of the textbook, and the way the textbook was used during the lectures seem to influence the role of the textbook as a learning tool. Possibilities and limitations were discussed within the given theoretical framework.

This study suggests that higher awareness about the assumed and real role of the mathematics textbooks at tertiary level is necessary. Some suggestions for further research that might provide deeper insights about the issue are given.

Sammendrag

Studenter på høyere nivå antas å jobbe intensivt og individuelt med nye matematiske begrep (Wood, 2001). I denne sammenheng kan lærebøker i matematikk være viktige verktøy for læring. Denne avhandlingen tar for seg spørsmålet om hvilke faktorer som påvirker matematikkbokas rolle som læringsverktøy.

Studiekonteksten er et kalkuluskurs for første-års ingeniørstudenter. En kort pilotstudie viste at de fleste studentene foretrakk å bruke forelesningsnotater framfor å bruke læreboka, selv om de i starten av semestret betraktet boka som et viktig læringsverktøy. Dette var utgangspunkt for studien i avhandlingen, som har som mål å utforske hvilke faktorer som muligens kan påvirke den rollen en lærebok kan ha som læringsverktøy for studenter som tar matematikk som ‘et service emne’.

Læreboka ble definert som et potensielt lærings-verktøy forankret i en pedagogisk setting der et grunnleggende kalkulus-kurs tilbys. Studiet ble gjennomført mens studentene jobbet med derivert-begrepet. Prosessen der studentene nærmer seg derivert-begrepet ble studert fra et epistemologisk, et kognitiv og et didaktisk perspektiv. Denne studie er et kasusstudie innenfor et fortolkende paradigme. Ulike data er samlet inn: Spørreskjema, observasjoner av forelesninger og oppgaveløsnings-økter, intervju med lærere og studenter samt informelle samtaler. I tillegg ble et spørreskjema sendt til forfatterne av de mest brukte kalkulus-lærebøkene med den hensikt å utforske deres visjoner om egne tekster.

Resultatene fra hovedstudien bekrefter de observerte fenomener i pilotstudiet. Studentene oppfattet læreboka som vanskelig og forelesningsnotatene ble foretrukket. Læreboka ble brukt hovedsakelig til å lese eksempler og finne mulige prosedyrer når studentene arbeidet med oppgaver. Funnene i studien viser både muligheter og begrensninger av epistemologisk, kognitive og didaktisk karakter. Muligheter og begrensninger ble diskutert innenfor det gitte teoretiske rammeverk. Studentenes svake forkunnskaper, deres tilnærming til å lære matematikk, den formelle tilnærmingen til matematikk i læreboka og måten læreboka var brukt i forelesninger påvirker bokas rolle som et verktøy for læring.

Denne studien tyder på at større bevissthet om antatt og reell rolle for matematikklærebøker på universitets- og høgskolenivå er nødvendig. Noen forslag til videre forskning som kan gi en dypere innsikt i problemet er gitt.

Contents

1 Introduction

1.1 Background for the study

When Narvik University College searched for a doctoral student I applied for the position. The area of study was already decided to be the mathematics textbook at tertiary level. The context of the study should be the basic mathematics course taken by first-year engineering students. The textbook used during the course should be in focus. The area of the study seemed to be very interesting. I assumed that textbooks used at primary and secondary level had a strong impact on teachers' instructional decision-making and that the teachers depended on the textbook. I read the PhD thesis of Monica Johansson (2006) and some of her conclusions confirmed my view. But what characterises the situation at the tertiary level? Mathematics at tertiary level is formally presented in textbooks and during the lectures. What role does the textbook have in the learning and teaching process? How is the textbook used by the students? And what factors might possibly influence the process of using the textbook? After some considerations and discussions with the supervisors I decided to focus on the learner that is supposed to use the textbook in a meaningful way. The first idea was to adopt a pure cognitive perspective and study how the first-year engineering students worked with the tasks. But my view on the process of students' use of the textbook was extended during the first weeks of observations and I realized that new perspectives had to be adopted. I will reveal more about this view in the next section.

1.2 The research process – a brief overview of the papers

The thesis consists of four papers and this overview putting them into a more general frame. Three of the papers are published in international educational journals and the fourth one is accepted for publication in *International Journal of Mathematical Education in Science and Technology*.

The papers are:

1. Randahl, M. & Grevholm, B. (2010). Learning opportunities offered by a classical calculus textbook. *NOMAD*, *15*(2), 5-27.
2. Randahl, M. (2012a). First-year engineering students' use of their mathematics textbook – opportunities and constraints. *Mathematics Education Research Journal*, *24*(3), 239-256.
3. Randahl, M. (2012b). Approach to mathematics in textbooks at tertiary level – exploring authors' views about their texts.

International Journal of Mathematical Education in Science and Technology, *43*(7), 881-896.

4. Randahl, M. (2016, accepted for publication in *International Journal of Mathematical Education in Science and Technology*). The mathematics textbook at tertiary level as a curriculum material – exploring the teacher's decision-making process.

The format of a collection of articles was chosen because of the wish to address the issue of textbooks at tertiary level to the mathematics education community, both in Norway and in other countries. In what follows I briefly present the research process with focus on how the research themes of papers were chosen and the research developed. It was inspired by Arcavi (2000) who says that "researchers usually present finished polished results and say little about the ways in which the projects started and developed" (p.1).

It was also given that the context of the study should be a basic mathematics course taken by the first-year engineering students and the focus should be on the students using the textbook. The textbook used in the course was *Calculus. A complete course* written by Robert A. Adams (2006). The basic mathematics course is compulsory for first-year engineering students and it usually consists of two parts: calculus and linear algebra. In this study only the calculus part was considered. As mentioned before, I intended to use a cognitive perspective on the study, focusing on how engineering students work with different tasks from the textbook and try to explore potential problem areas. But I still did not know what was really important and how I should approach the study. Schoenfeld (1999) says:

> The hard part of being a mathematician is not solving problems; it is finding one that you can solve, and whose solution the mathematical community will deem sufficiently important to consider an advance… In any real research (in particular, education research), the bottleneck issue is that of problem identification - being able to focus on problems that are difficult and meaningful but on which progress can be made (as quoted in Selden and Selden, 2001, p. 239).

In order to sharpen the focus I decided to conduct a short pilot study. The study was conducted during four weeks in autumn 2006. The used methods were observations of the lectures, and some informal talks with the students and the teacher. During the observations I realised that the students' use of the textbook was altered in some way. In the beginning of the observation period nearly all students following the lectures had the textbooks on the desk in front of them. Many students opened the books on the pages where the actual topic was discussed. They made lecture notes and looked frequently into the textbooks. I asked some of them (informal talks during the breaks) about the reasons for looking into the textbook. They answered that they wanted to check if and how the lectures were related to the textbook.

Excerpts from two informal talks:

Conversation 1

My question: I noticed that you frequently looked at the textbook during the lecture. For what reason?

Student 1: I wanted to see if I had to make detailed notes….I mean if I could find the same in the book…

Student 2: I looked at the sequencing of the topic in the textbook….is it the same as the sequencing on the blackboard….Is anything missing? ……

Conversation 2

My question: I noticed that you frequently looked at the textbook during the lecture. For what reason?

Group of the students (one student answers, 3 others students agree)

Student: I wanted to check if there was more in the textbook, if I had to read more later, …. at home…

During the next weeks of the pilot study I noticed continuously decreasing interest for the textbook. The students followed the lectures and made notes. The textbooks were still placed on the desks in front of them but they were usually closed. But when working with tasks, the students seemed to prefer using the lecture notes rather than the textbook.

Excerpt from informal talk:

My question: What do you prefer to use when working with math? Notes or textbook?

Student: Notes,…they are easier to read,…..you know, I can better understand what is going on. Easier to use when solving tasks…..

I asked also the teacher if it was some kind of known phenomenon that many students preferred to read and use the notes.

The teacher said: “I think that many of the students combine using the textbook and the lecture notes. Maybe the most clever students use the textbook a little more than the others,….those with problems, you know. I try to encourage them to use both. I assume that the textbook and the notes are in some ways complementary …so they should use both”.

I perceived the decreasing interest for the textbook as an interesting phenomenon and became curious about it. Sierpinska and Kilpatrick (1998, p. 16) state that an important aim of all research should be to “satisfy the curiosity of the researcher about some situations”. And further “the curiosity should lead to an understanding of situations” (p. 16).

The conclusion after the pilot study was: The observations of lectures, task-solving sessions and some informal talks during the pilot study indicated that the students gradually seemed to change their perception of the textbook as less useful than the lecture notes. This could possibly mean that the students had some problems concerning the use of the textbook when working with mathematics. The following hypothesis was formulated: There are possible factors influencing students’ approach to the textbook and causing their difficulties to use it.

Assuming that the textbook should be used as a learning tool, I wanted to get an understanding of what could possibly cause the students' reaction. More generally: What could possibly influence if and how the students use the textbook when learning mathematics.

Thus the focus came to be on answering the question: What are the opportunities and constraining factors in the process of approaching the textbook as a learning tool?

I assumed that the students' motivation for taking the course was that they wanted to learn mathematics in order to get knowledge necessary for future more specific engineering courses and for passing the exam. The textbook should help them to achieve the goals. The comments from the teacher suggested that maybe the cleverest students used the textbook more fluently than other students. So students' abilities to learn mathematics might possibly influence the way of perceiving and using the textbook. But I wanted to identify and explore other reasons that might be significant as well. What about the textbook itself? Was the chosen textbook the 'right' one for the course and for the particular group of students? And what does it mean to be 'the right' book? The educational setting might also influence the perception and use of the textbook. How was the textbook embedded in the context of the course offered by the particular educational institution? How was the textbook used by the teacher? It seemed necessary to take into account different perspectives when considering the role of the textbook as a useful learning tool.

Summing up: I decided to use a holistic perspective and explore the process of approaching the textbook by the students as a learning tool. The aim was to identify possible opportunities and constraints that might influence the role of the textbook. To achieve more knowledge about the opportunities and constraints might help to understand and explain why the textbook could/could not play the expected role as a potential learning tool. I started to search research literature with focus on the following key issues: learning calculus, engineering students, mathematics textbooks at tertiary level, learning about the derivative concept and teaching practice at tertiary level. I decided to conduct a study of how students approach the mathematics textbook and to identify which factors come into play when students attempt to use the textbook. The characteristics of the textbook as a learning tool had to be identified prior to the observational work. The focus could be subdivided into the factors that were imposed by the nature of the knowledge in the textbook, characteristics and beliefs of the students and the teaching practice in which the textbook was supposed to be embedded. The investigation of the process of approaching the textbook was situated within three different perspectives associated with constraints linked to "the epistemological nature linked to the mathematical knowledge at stake," the "cognitive nature linked to the population target by teaching,” and the “didactical nature linked to the institutional functioning of the teaching” (Artigue, 1994, p. 32). I assumed that using these perspectives was conducive to getting an overall picture of the process of approaching the textbook, and exploring the influencing factors.

The conclusion before the main study was: I wanted to get better understanding of the role of the textbook as a learning tool in the context of the basic mathematics course for first-year engineering students. What is this role possibly dependent upon? The study attempted to use empirical methodology in order to identify possible opportunities and constraints in the process of approaching the textbook as a learning tool. The epistemological, cognitive and didactical perspectives were to be used.

The new group of students started their mathematics course in January 2007. A questionnaire was distributed to all students initially and I started with observations of lectures and some task solving sessions. I also had many informal conversations with the students. The students finished their work with chapter 1 *Limits and Continuity* in the textbook and started to work with chapter 2 *The Derivative*. The study was conducted when they were working with early treatment of the derivative concept.

At the same time I started the text analysis of the calculus textbook prescribed for the students. The text analysis was done in cooperation with my supervisor Barbro Grevholm. The three first sections of the chapter “Differentiation” were studied. The analyzed content was similar to the subject matter of the lectures and task solving sessions during the observation period. Some of the results of previous research revealed the engineering students’ tendency to perceive mathematics mostly as a collection of procedures useful in the engineering context. It seemed reasonable to consider what the textbook offered the first-year engineering students. I wanted to explore how the textbook might assist or hinder the students in their meaningful learning of the derivative concept. The focus was on the introduction of the derivative concept, the role of the definition and on how the book promoted procedural and conceptual knowledge. The first paper *Learning opportunities offered by a classical calculus textbook*, presents the results of the study.

The three perspectives (Artigue, 1994) were introduced in the second paper *First year engineering students’ use of their mathematics textbook – opportunities and constraints*. They were used in order to identify possible factors influencing the process of approaching the textbook as a potential learning tool.

The mathematical textbook is a result of some visions and work done by the authors. Exploring authors’ ideas about their texts could possibly illuminate the students’ perceptual problems with the textbook as being difficult and of marginal use. Thus, a questionnaire was designed and sent to seven authors of the most used calculus textbooks in the Scandinavian countries and USA. Four authors responded. Their answers were analyzed in relation to results of previous research about students' difficulties with calculus. The results and discussion are presented in the third paper *Approach to mathematics in textbooks at tertiary level – exploring authors' views about their texts*.

The fourth paper *The mathematics textbook at tertiary level as curriculum material – exploring the teacher's decision-making process* focusses on the teacher's decisions and use of definitions, examples and exercises in a sequence of lectures about the derivative concept. The introduction of the derivative concept proposed by the teacher during the lectures was analysed in relation to the results of content text analysis of the textbook (Randahl & Grevholm, 2010). The teacher's decisions in the present study were explored through the lens of intended learning goals for first-year engineering students taking a basic mathematics course.

1.3 The research questions

As stated above the research attempts to identify and explore the factors that come into play when the first-year engineering students approach their mathematics textbook as a potential learning tool. The term learning tool will be explained in the section about the theoretical background for the study. The overall research question became:

> What are the opportunities and constraints when first-year engineering students approach the mathematics textbook as a potential learning tool?

The questions in the articles were posed in order to gain insights to the overall research question and they are related to the aims of the specific articles.

Article 1 (Randahl & Grevholm, 2010) with aim: to examine as an entirety what students are offered by the book to learn about the concept of derivative. Research questions:

1. What characterizes the introduction of the derivative and the further treatment of the concept in the calculus textbook for first year engineering students?
2. What kind of knowledge does the textbook emphasize?

Article 2 (Randahl, 2012a) with aim: to investigate students' approach to the mathematics textbook from epistemological, cognitive and didactical perspective. Research questions:

1. What characterizes first-year engineering students' approaches to mathematics textbooks?

2. What possible opportunities and constraints might influence the ways the textbook are approached by the students?

Article 3 (Randahl, 2012b) with aim: to investigate the textbook authors' views about their texts. Research questions:

1. What characterizes authors' vision of the calculus textbook offered to the first year students?
2. What characterizes authors' views about the introduction of new mathematical concepts?
3. In what ways do these views correspond with the results of previous research?

Article 4 (Randahl, 2016) with aim: to explore how the textbook is used by the teacher during the lectures. Research questions:

1. To what extent does the teacher adopt the textbook's approach to introduction and early treatment of the derivative concept during the observed lectures? What modifications, if any, are made?
2. How does the selection of examples and exercises fit with the learning goals for first-year engineering students taking a basic mathematics course?
3. How are the learning opportunities/constraints (as pointed out in the content text analysis) of the textbook utilized/overcome by the teacher during the implementation?

The overall research question addresses the main aim of the research. The question comprises the implicit assumption that some factors influencing the process of approaching the textbook exist and that they can be identified. In order to gain insights to the overall research question and answer it some more specific research questions had to be posed. The specific questions were developed during the study. In all the questions the inquiry was directed towards the process of approaching the textbook with focus on the learner and her/his needs. The research questions in Article1 attempted to explore what the textbook has to offer in order to learn mathematics by the first-year engineering students. The results of the article were taken into account when the further questions were posed. The idea of exploring calculus textbooks authors' views about their texts was found interesting and the research questions according to this aim were formulated. The questions in Article 3 were posed on the basis of both the theory used in Article 1 and Article 2, and also on the basis of the results that emerged, particularly from the analysis of the textbook in Article 1.The epistemological, cognitive and didactical perspectives were explicitly described in Article 2 but the perspectives were decisive when deciding the aims and posing the questions in both Article 3 and Article 4. Since the research questions guide the decisions about research design and the collection and analysis of the data, it is crucial to consider the nature of the research questions (Bryman, 2004). The investigation in this study attempted to identify and explore the factors that might influence the role of the textbook. The phenomenon of students' change in how they perceived the textbook's importance when learning calculus should be examined. In order to state that some factors are opportunities and/or constraints in the process of approaching the textbook some relationships should be indicated. It implies that the research has both descriptive and exploratory aspects. This is illuminated in the nature of the posed questions. For example the question *What characterizes first year engineering students' approach to mathematics textbooks*? focuses on the issue of what things are while the question *What possible opportunities and constraints might influence the way the textbook is approached by the students?* focuses on what factors that appear to be important when exploring students' approach to the textbook.

2 The engineering education setting

2.1 Engineering education in Norway

The engineering education is one part of the higher education system and the responsibility for it lies with the Norwegian Ministry of Education and Research. In accordance with the Bologna process the national higher education system in Norway is a 3+2+3 year system. Acceptance to higher education requires fulfilled three years of upper secondary school with general university admissions certification. Additionally, study competence can be achieved by the so-called 23/5 rule where applicants must be 23 years of age and have a total of five years of upper secondary education and work experience as well as have passed courses in Norwegian, English, mathematics, science, and social studies. To be accepted at a certain educational programme (like for instance engineering) advanced courses in mathematics, physics and chemistry must be passed. The curriculum as a core of any educational system describes what learners have to accomplish. The teachers use their subject-matter backgrounds and pedagogical aims to achieve the goals of the curriculum. The mathematics course for first-year engineering students is compulsory and a certain level of mathematical knowledge is expected.

In 2003 a national reform, called the Quality reform was implemented in the educational system in Norway. Its intention was to improve the quality of higher education and it resulted in more interest and focus on issues related to mathematics learning and teaching. However, there is still a certain need for research about tertiary mathematics education, for example there is a need to know more about how tertiary mathematics is taught to students enrolled in mathematics service courses. The last evaluation of Norwegian degree programmes in engineering (2006-2008) was performed by NOKUT (Norwegian Agency for Quality Assurance in Education). The aim of the evaluation was “to provide the best possible basis of information and expert opinion for further development of the study programmes”. In the evaluation some critique for having too high attrition was given, and the following areas were pointed out as ‘requiring special initiatives to raise standards’: educational skills of the teaching staff, the lack of research-based teaching, and internationalisation (NOKUT, 2008). It was expected that the institution offering the program took into account the recommendations about improvement.

2.2 Mathematics for engineering students

Mathematics is generally viewed as an important subject in engineering education. However, when analysing the issue of mathematics for engineering students it is essential to pose and reflect on the question: What mathematics do the engineering students need to learn? According to the core curriculum in the department where the study was conducted, one of the aims for taking the mathematics course was: to obtain mathematical knowledge and skills in order to use them in identifying, analysing and solving engineering problems (for more details see Randahl, 2012a). The aim pointed out the importance of understanding of the main mathematical concepts in order to use them when solving problems in an engineering context.

When considering the calculus part of the basic mathematics course for engineering students, it has to be taken into account that calculus has many formal aspects. The formal definitions are frequently used and some proofs are presented (Tall, 1991). Thus the following questions arise: Do the engineering students need to focus on the formal mathematics? Or maybe they need mostly focus on applied mathematics? Kümmerer (2001) points out that mathematics is interplay between the abstract and the applied foundations and that it has consequences for what mathematics should be offered to the students. He notices:

> Most of mathematics emerged directly or indirectly from applications…..Many real word problems require a sort of mathematics that is still too difficult for the mathematicians…..but mathematics itself seems to resist any division into an applied part and a pure part. It follows that students have to be taught mathematics and how to apply it (p. 325).

However, existing research suggests that students struggle with making sense of formal definitions and grasping definitions' notations (Tall & Vinner, 1981; Vinner, 1991; Cottrill, Dubinsky, Nichols, Schwinkendorf, Thomas & Vidakovic; 1996). Previous research proposes notions and approaches to help students in more successful concept formations. According to Cornu (1991) the students have to perceive the concept as a useful tool when solving problems in particular contexts. Tall and Vinner (1981) proposed the notion of concept definition and concept image in order to emphasise the importance of development of rich concept image by the learner.

Previous research reveals some problems that the engineering students experience when applying mathematics knowledge in engineering contexts. Mathematics courses provide students with knowledge that make it possible to perform well in their mathematics classes, but do not necessarily prepare them to use the knowledge successfully in their engineering classes. Jones (2010) in his study points out the demands when applying mathematics:

> In physics or an engineering course, problems are often presented in real world contexts, using words, figures, and tables to organize and communicate the situation to be solved. Students are expected to take these situations and to create mathematical equations from which they can perform procedures. Students also need to dissect equations and to describe relationships between multiple variables (p. 2).

Also Selden and Selden (2001) discuss the issue of complaints from client disciplines, such as engineering, that the students cannot apply the supposed mathematical knowledge. However, it is not obvious that more focus on applications does solve the problem. One requirement for success when working with mathematical tasks in new contexts is “a solid understanding of the mathematical concepts involved” (p. 10).

To understand the idea behind the mathematical concept is important when learning calculus. According to Apostol (1967, p.1):

> Calculus is more than a technical tool – it is a collection of fascinating and exciting ideas that have interested thinking men for centuries. These ideas have to do with speed, area, volume, rate of growth, continuity, tangent line, and other concepts from a variety of fields. Calculus forces us to stop and think carefully about the meanings of the concepts

The notions of conceptual knowledge and procedural knowledge are essential when considering learning mathematics by engineering students. Both interviews with and observations of students and teachers in the engineering programme (Randahl, 2012a) showed that the students focused heavily on algorithms and procedures when working with mathematical problems. The relationship between concepts and procedures is an important issue in learning of mathematics (Hiebert & Lefevre, 1986; Silver, 1986). Procedures that are learnt with meaning are linked to the conceptual knowledge. Students with only procedural knowledge can obtain correct answers when working with problems, but they do not understand why they use a specific procedure. On the other hand, students with understanding of mathematical concepts can have problems with using procedures. Thus both the conceptual understanding and procedural fluency are important.

When learning new mathematical concepts the importance of the process of making abstractions is pointed out by Kümmerer (2001). He claims:

> The universal applicability of mathematics relies on its abstractness: Going from the concrete to the abstract means at the same time to increase the number of situations where structural parallels can be recognized. It thus increases the range of possible applications. At the other side of the same coin, abstraction decreases the number of facts to be handled separately and thus forms an important part of what is called ‘understanding’. Therefore, it is of special importance to let students experience the power of abstraction (p. 325).

I will come back to the issue of making abstractions when discussing the concept of derivative in the next chapter.

In an attempt to draw some conclusion regarding what aspects of mathematics that are important for first-year engineering students who take the mathematics course, I assume that the engineering students need:

- To understand the ideas behind and the meaning of the main mathematical concepts (Apostol, 1967) in order to see the relation of mathematics to the intended applications (Selden & Selden, 2001). The experience of abstractness is valuable (Kümmerer, 2001).

- To achieve conceptual understanding of the concepts and procedural fluency in order to obtain answers to the problems (Selden & Selden, 2001).
- To achieve understanding of the formal definitions of mathematical concepts embedded in the idea of the concept.

The issue of engineering students' needs will be further discussed when the concept of derivative is considered in the next chapter.

3 The derivative concept

The mathematics context of this study is the derivative concept. In this chapter some results of previous research about the derivative are outlined. Examples of how the derivative is introduced and treated in the calculus textbooks are presented. Finally some aspects of the introduction of derivative concept for engineering students are considered.

3.1 Previous research about learning of the derivative concept

Many studies within mathematics education research focus on students' difficulties with calculus concepts. Students have a procedural but not conceptual understanding of calculus (Tall, 1992a, 1992b). White and Mitchelmore (1996) show that students have problems with making sense of basic calculus concepts.

The derivative is one of the fundamental concepts in calculus and its understanding constitutes foundations for both other concepts and for the work with differential equations. The derivative reveals how fast the function value $f(x)$ changes according to the change of x for a given value of x. It is the *local aspect* of the concept. The global aspect of the derivative focusses on the derivative as a new function. The concept is usually approached in two different ways: as a slope of a curve (geometrical approach) or as a rate of change (physical approach).

The concept of derivative is complicated because it relies on other concepts like function, tangent, and limit which create many problems for the students (Orton, 1983; Cornu, 1991; Zandieh, 1997; Juter & Grevholm, 2007). For example students often have an idea about the tangent line that is created by the tangent of a circle. It means that they think about the tangent line as a line that meets the curve at one point and may not cross the curve at this point (Biza & Zachariades, 2006). It might create problems when students meet the derivative concept presented by use of the geometrical approach. The study of Viholainen (2008a) shows students' problems with informal and formal understanding of the concepts of derivative and differentiability. Students are not able to connect formal and informal reasoning and avoid using the formal definition of the derivative in problem solving situations.

There is not much empirical research about how the engineering students learn the concept of derivative. However, some results have been achieved. The study of Bingolbali, Monaghan and Roper (2007) explores engineering students' conceptions of and preferences for derivative. The study also focuses on engineering students' views about mathematics. The results show that engineering students prefer the rate of change aspect while mathematics students' conception develops in the direction of tangent aspect. The study also shows that engineering students view mathematics as a tool. As a result of this view they expect the application aspects to be emphasised in their mathematical courses.

3.2 The derivative concept in textbooks

The mathematics textbook at tertiary level has a long tradition. Many mathematicians regarded Euclid's *Elements* as an example of how mathematics knowledge should be presented as a text. This fundamental work consisted of 13 books written by the Greek mathematician Euclid in Alexandria around 300 BC. The first textbook in the differential calculus was the *Analysis of Infinitely Small Quantities for the Understanding of Curves*, and it was written by Guillaume Francois l'Hospital in 1696. The text provided a treatment of the calculus of differentials and its applications and the book was very successful (Katz, 2004). The most influential calculus textbooks in the eighteenth century were the three works of Euler: *The Introduction to Analysis of the Infinitive* of 1748, the *Methods of the Differential Calculus* of 1755, and the *Methods of the Integral Calculus* of 1768 – 1770 (Katz, 2004, p. 350). The books presented an organization and a clear explanation of the material that was developed before Euler's time. The textbooks were used mainly in the private education of students. But the historical events influenced the development of mathematical texts. As Katz says

> It was however, the new students entering the sciences after the upheavals of the French revolution who inspired the writing of many new texts, which replaced those of Euler and were the direct ancestors of the texts of today (p. 354).

More currently used textbooks approach the introduction and treatment of calculus concepts in different ways. Some of the older textbooks, for example the one by Apostol (1967), followed the historical development of calculus and treat integration before differentiation. The majority of the recently used calculus textbooks introduced the derivative concept first, followed by the concept of integral.

In the calculus textbook used in this study (Adams, 2006), the chapter about derivative is proceeded by the chapter about the limit concept. The derivative is introduced as the limit of the Newton quotient secants. When the global aspect - the derivative as a new function - is emphasised, the local aspect, the derivative of the function at a fixed number, is treated more implicitly (for more details about introduction and treatment of the derivative see Randahl and Grevholm, 2010). In another, widely used calculus textbook, written by Stewart (2003) first the concept of the derivative in a point is presented. Then the concept from a single point is extended to the derivative function f'. These two aspects of derivative are outlined as follows:

…we considered the derivative of a function *f* at a fixed number *a*:

$$f'(a) = \lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$$

Here we change our point of view and let the number *a* vary. If we replace *a* in Equation 1 by a variable *x*, we obtain

$$(1) \quad f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

Given any number *x* for which this limit exists, we assign to *x* the number $f'(x)$. So we can regard f' as a new function, called the derivative of *f* and defined by Equation (1), (p. 165).

The presentation proposed by Stewart makes the two aspects of the derivative concept more clearly explained. Both the book by Adams and the book by Stewart have been and are still used as textbooks for calculus courses for first-year engineering students in Norway. The approach to the introduction of the calculus concepts used by the textbooks is formal. As noticed by Lakatos (1976) it might create problems for the students.

> This style starts with a painstakingly stated list of axioms, lemmas and/or definitions. The axioms and definitions frequently look artificial and mystifyingly complicated. One is never told how these complications arose (p. 142).

Some studies consider the nature of tasks proposed in the calculus textbooks (Lithner, 2004; Randahl & Grevholm, 2010; Raman 2004). The results show that the majority of tasks are of algorithmic character and that many of them can be solved by imitation of the examples of the textbook. The study by Randahl and Grevholm (2010) shows, that the textbook of Adams proposes many tasks that promote conceptual understanding. However, the location of these tasks at the end of a task sequence might be a constraint since the students do not always have time to work with them. It emphasises that it is important to consider how the textbook is embedded in the teaching practice (Randahl, 2016).

However some selected textbooks take into account students' difficulties with grasping the idea behind mathematical concepts and propose different approaches to introduce the concept. An example of using other approaches to introduce the derivative is found in the textbook *Number and Functions: Steps into Analysis* written by R. P. Burn. Alcock and Simpson (2001) wrote the following about the approach:

> The text consists mainly of questions, with compressed solutions and a very short summary of the main ideas at the end of each chapter. The questions develop rationales for the main definitions and construct the central arguments that lie behind the main theorems in subsequent arguments (p. 101).

Another example of a textbook that proposed a different approach when introducing the calculus concept is 'Mathematics for decision making' by Martin (1969), for more details see Grevholm and Randahl (2010).

3.3 The derivative concept for engineering students

When discussing the derivative concept in the context of the mathematics course for first-year engineering students the issue of needs for this particular group of students has to be considered. What do the future engineers need to learn about the derivative and what approach to the introduction of the concept is appropriate?

The curriculum for mathematics for first-year engineering students (Randahl, 2012a) states that the students have to obtain an understanding of mathematics concepts in order to use them when solving engineering problems.

Generally, solving problems with the use of mathematics implies that the problem posed in a context has to be mathematised.[1] A selection of relevant objects and relations and the choice of appropriate mathematical representations have to be made. Mathematical methods have to be used in order to achieve mathematical answers. Finally the results must be interpreted in the initial context of the problem (Blomhøj & Højgaard, 2003). The process, usually called modeling, requires an understanding of the reality of inquiry and the ability to choice mathematical concepts that describe the reality and use them in a proper way. It means that the understanding of ideas behind different concepts (Apostol, 1967; Selden & Selden, 2001) and of the relationship between them is necessary when approaching a problem. This calls on conceptual knowledge. The identification of the appropriate calculus concepts and utilization of relationships between them requires conceptual knowledge (White & Mitchelmore, 1996; Tall, 1991). Further the identified concepts and possibly relations have to be expressed in symbolic form. This requires the knowledge of different representations of the concept and the ability to switch between them. To complete the solution, procedural fluency is needed (Hiebert & Carpenter, 1992).

Summing up: The application of any mathematics concept when solving problems in context requires understanding of the idea behind the concept and achieving conceptual and procedural knowledge.

When relating the considerations above to the derivative concept the following questions arise: What do first-year engineering students need to learn about the derivative in order to effectively and successfully complete tasks in context? And what features of the derivative concept should be focused? In the following the idea behind the derivative, different representations of the derivative, and conceptual- and procedural knowledge related to the derivative will be considered. The considerations will be connected to the issue of the appropriate introduction and treatment of the derivative for first-year engineering students.

[1] It is assumed that the problem is already formulated.

When solving problems involving the derivative the students have to be able to recognize the derivative in the posed context and then represent it by means of symbols and relational representations. This requires the understanding of what the derivative is about. The idea behind the derivative is change and rate of change. Many problems in reality which focus on the situations when a change in one variable affects other variable give rise to the idea of derivative. For example studying *the change in distance* (or *velocity*) of an object with respective *change in time* provides two situations for derivative - *velocity* if the function is *distance* and *acceleration* if the function is *velocity*. Or it could be studying how the power output of a generator varies with its temperature. By using the application aspect of the derivative the need of introducing the concept is emphasized. It might create more motivation and interest for working with the concept for engineering students. To focus on the rate of change aspect instead of the tangent aspect could be also appropriate for the engineering students since previous research identified engineering students' preferences when approaching the derivative (Bingolbali, Monaghan & Roper, 2007).

As shown in the previous section (3.2) the derivative concept in the textbooks is presented in a rather formal way. The formal definition of the derivative is based on the limit concept that is known to be difficult for the students. In the above examples the average rate of change can be calculated over different intervals and it creates the need for limiting process of difference quotient. Thus the formal definition of derivative will arise in natural way (Hähkiöniemi, 2006). By working with different examples where the focus is on the phenomenon of change, the students can get opportunity to extract the concept of derivative. According to Kümmerer (2001) it is an *act* of abstraction that should be emphasized in the course for engineering students. Abstracting is understood here as a mental activity by which the learner becomes aware of similarities (Skemp, 1986).

When introducing the derivative concept, both the local perspective (the derivative of a function at a fixed value) and the global perspective (derivative as a new function) have to be emphasized. Some textbooks point out the global perspective and treat the local perspective in an implicit way (Randahl & Grevholm, 2010). Context problems usually are modelled by a function f. The graph of f' gives both quantitative and qualitative information about function f .

Generally, the use of multiple representations of mathematical concepts is expected to increase students' understanding (Tall, 1996; Duval, 2006). When considering the understanding of mathematics concepts Hiebert and Carpenter (1992) emphasize the importance of seeing the connection between ideas, representations and procedures. Different aspects of the derivative concept may be express by various representations. The derivative can be seen (a) physically as speed or velocity, (b) verbally as the instantaneous rate of change, (c) graphically as the slope of the tangent line to a curve at a point, and (d) symbolically as the limit of the difference quotient. The different representations of the derivative might be used to see the meaning of the formal definition of the derivative. Within every representation the derivative can be seen as a function whose value at any point is the limit of the ratio of differences.

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

This might help the students to achieve understanding of the formal definition of the derivative.

It is also important to use interchangeably different notations of the derivative such as

$f'(x), \frac{df}{dx}, \frac{d}{dx} f(x)$ since different books prefer different notations. This also gives the opportunity to discuss the advantages and difficulties with the use of different notations, for example in relation to composite functions.

Summing up: To treat the concept in correct and effective ways in different situations the engineering students need to be able to have conceptual and procedural knowledge. The students need to be able to use the symbolism meaningfully to formulate (later in the course) differential equations and to be able to solve them. Procedural knowledge makes it possible to use particular rules and procedures within relevant representations of the derivative. It makes it possible to get an answer to the posed problem. Considering the goals for engineering students' learning Jaworski (2009) recognizes the importance of both kinds of knowledge by saying: "I want all students to be able to engage with mathematical concepts, to develop both conceptual understanding and procedural fluency and to be able to apply these to their engineering tasks" (p. 1590). Thus engineering students have to achieve the procedural fluency when treating the concept of derivative, but the conceptual understanding of the derivative is crucial.

4 Theoretical background

In this chapter first the view on learning mathematics as adopted in this study is presented. Then I explain what I mean when considering the textbook as a learning tool. Further the three perspectives (Artigue, 1994) are outlined. For the purpose of analysing the results the term affordance has proven to be useful.

4.1 Conceptualisation of the textbook

4.1.1 Learning mathematics

The process of learning mathematics is viewed from a social constructivist perspective. Social constructivism as a learning theory can be characterised by two features as stated by Ernest (1991):

> First of all there is the active construction of knowledge, typically concepts and hypotheses, based on experiences and previous knowledge. These provide the basis of experiences and previous knowledge, which in turn provide the basis for understanding and serve the purpose of guiding future actions. Secondly there is the essential role played by experience and interaction with the physical and social worlds, in both physical action and speech modes (p. 95).

I understand the learning process as building up knowledge by the individual learner in interaction with others. The learner constructs a meaning of the world that depends on her/his beliefs, needs, previous knowledge and social interaction.

Further, Ernest talks about a social constructivist account of mathematics education:

> A central thesis of social constructivism is that the unique subjective meaning and theories constructed by individuals are developed to 'fit' the social and physical worlds. The main agency for this is interaction, and in the acquisition of language, social interaction (p. 105).

The social constructivist view on learning takes into account that human beings are formed through their interactions with each other as well as by their individual processes. Social constructivism is built on the idea that social interaction and negotiation can help learners in their knowledge acquisition process. In this perspective both individual reasoning and social processes have central and essential parts to play when learning mathematics.

Also Björkqvist (1998) emphasizes that constructivist conceptions of learning assume that knowledge is individually constructed and socially co-constructed by learners based on their interactions in the world. The meaning that learners construct depends on their needs, beliefs, and prior knowledge. Related to learning mathematics at tertiary level, the individual constructs her meaning based on own previous knowledge and in response to experiences in social contexts as for example lectures, task solving sessions or discussions with the teacher. Further, from the constructivist perspective, teaching and learning focus on learner's investigation of 'mathematical unknowns' instead of teacher delivery of 'mathematical knowns' (Malone & Taylor, 1993). Related to use of mathematics textbooks in the learning and teaching process inquiry and discovery are valued instead of definitions, theorems and proofs.

4.1.2 The textbook as a learning tool

In the process of approaching the textbook as a potential learning tool, as understood in this study, the perception and initial use of the textbook by the learner are included. When perceiving the text in the textbook the students make some initial evaluations about what kind of learning the textbook offers and what possible difficulties can arise. Students also evaluate the relevance of the knowledge in the textbook in relation to their own learning goals.

The mathematics textbook is the main instructional material that the students traditionally use when learning mathematics (Love & Pimm, 1996). The textbook was conceptualized in different ways in previous literature and research. For example Stray (1994) proposes the following definition for the textbook: " […] a book designed to provide an authoritative pedagogic version of an area of knowledge" (p. 2). Johansson (2006) explores how teachers used the textbook in their classrooms, perceiving the textbook as the *potentially implemented curriculum*. Otte (1986) emphasizes the communication aspect saying that the textbook is "produced by human being for the purpose of communication" (p. 175). From a socio-cultural perspective the textbook is regarded as an artefact that is historically developed, culturally formed, produced for certain ends and used with particular intention (Rezat, 2006).

For the purpose of this study, the textbook is conceptualized as a *learning tool* embedded in an *educational tertiary setting.* It means that the textbook is assumed to be used by the students in a learning process in particular learning environments provided by an educational setting. By a *learning tool* I mean a cognitive tool that promotes cognitive processes related to meaningful learning of mathematics. The concept of cognitive tool has been used previously in the area of computers (Dreyfus, 1994; Jonassen, 1995). Similarly, as pointed out when regarding the computer tools, the textbook should support and guide the learner. In this study the role of the textbook is viewed not only as a 'facilitator of knowledge acquisition' but mainly as influencing the process of knowledge construction and assisting the learner in the learning process. The active role of the learner is essential. The learner shall learn with the cognitive tool rather than from it (Jonassen, 1995). Considering the learning opportunities provided by the textbook I am aware about both the learning opportunities and the cognitive demands of the textbook. The issues of epistemological change of the knowledge from lower to tertiary level (Tall, 1991) and the concept formation are of interest. Considering the textbook as a learning tool for the engineering students the issue regarding engineering students' needs arises. The conceptual and procedural knowledge are significant in these considerations. Similarly as pointed out when regarding the computer tools, the idea of using several representations of the same concept is important when considering what the textbook offers for learning. Visualization helps the learner to construct mental images of the concept. Examples and tasks which stress different aspects of the concept are especially valuable for the learner.

To further clarify the term of *the textbook* as a *learning tool,* I attempt to define a priori some criteria. They are embedded in the view of learning from the social constructivist perspective. It means that the learner actively constructs the knowledge and that the meaning that the learner constructs depends on her/his needs, beliefs and prior knowledge. When viewing the learning from the social constructivist perspective, the issues of previous knowledge and concept formation are significant. It is also assumed that the approach to mathematics provided by the textbook and the way of using the textbook by the teacher take into account research results within mathematics education.

The textbook is potentially embedded in an educational setting. Thus the features of the setting, for example how the textbook is used by the teacher, have to be taken into account when considering the textbook's role. The textbook is not an independent object but part of the learning environments created by the educational setting. As Mogens Niss said (as a comment during the 90% seminar for this study): "We can only say if the textbook is good or not if we know what kind of setting the textbook is potentially embedded in". Considering the educational setting in this study the main focus is on how the teacher perceived and used the textbook in own teaching practice.

When considering the textbook as a learning tool the following criteria are proposed:

1. The textbook provides problems, examples and tasks that promote cognitive processes related to learning mathematics.
2. The approach to mathematics and learning mathematics proposed in the textbook is based on the results of research about learning and teaching mathematics.
3. The textbook is embedded as a learning tool in the institution where the teaching is offered and the learning process takes place. Since the textbook is intended to be used by the engineering students, the learning goal for this specific group of students should be focused on when choosing and using the textbook in the teaching process. The learning environments should promote students' active use of the textbook in which mathematical ideas are ex-

plored and both conceptual and procedural knowledge are focused on.

4.2 Three perspectives on the process of approaching the textbook as a learning tool

The study attempts to place the investigation of the process of approaching the textbook within three different perspectives associated with constraints linked to "the epistemological nature linked to the mathematical knowledge at stake," the "cognitive nature linked to the population target by teaching," and the "didactical nature linked to the institutional functioning of the teaching" (Artigue, 1994, p. 32). The different perspectives are chosen in order to get a holistic picture of the process under investigation. They are viewed as the lenses that make it possible to consider the different aspects of the process of approaching the textbook by the students. The notions and theories that help me to understand and make sense of the empirical data are outlined within each of the perspectives.

4.2.1 The epistemological perspective

Epistemology addresses the issue of knowing and learning. In this study the concepts of knowing and learning are related to learning and teaching of mathematics. The epistemological perspective when considering the process of approaching the textbook in order to learn mathematics refers to the following issues:

- The nature of the target mathematics knowledge in the calculus course and how the knowledge appears in the textbook.
- Students' views of the nature of mathematical knowledge.
- Students' views of the nature of learning; what it means to learn mathematics and how learning emerges.
- Teacher's view of the nature of mathematics knowledge that the engineering students taking calculus course need.

There are contrasting epistemological perspectives of mathematics like for example the absolutist view or fallibilist view (Ernest, 1991). The absolutist view describes mathematics as universal, certain, objective and established by proof. The fallibilist view describes mathematics as incomplete, changing and invented, rather than discovered. The issue of relation between epistemology and learning and teaching of mathematics has been discussed in educational research. For example Dossey (1992) stresses: "Mathematics educators need to focus on the nature of mathematics in the development of research, curriculum, teacher training, instruction, and assessment as they strive to understand its impact on the learning and teaching of mathematics" (p. 46). The change of nature of mathematical knowledge from secondary to tertiary level has been emphasised in the literature. At lower levels the mathematical knowledge is embedded in specific contexts and situations (Steinbring, 1998). Tall (1991) describes the move from elementary to advanced mathematics as a transition “from describing to defining, from convincing to proving in a logical manner based on those definitions” (p. 20). Mathematical knowledge at tertiary level is mainly based on the sequence: definition, theorem and proof (Tall, 1991). Mathematical definitions have a significant role in the introduction of new concepts (Vinner, 1991). The view of mathematics has consequence for how mathematical knowledge is presented in the textbooks (Lakatos, 1976). Mathematics in many classrooms and textbooks are based on assumptions like: concepts are mainly acquired by means of their definitions, definitions should be minimal and the students will use definitions to solve problems and prove theorems (Vinner, 1991). The analysis of textbooks conducted by Raman (2002) points out epistemological aspects of the role of the definitions when presenting new concepts.

Learning mathematics has an epistemological dimension by for example the fact that individual epistemological beliefs influence the learning process (Hofer, 2001). The social constructivist perspective on learning implicates that the issue of personal conceptualisation of knowledge and learning is significant. Schoenfeld (1992) emphasises that students’ ideas and epistemological conceptions of mathematics establish a psychological context for what it means to know and learn mathematics.

Previous research suggests also that students’ conceptions of mathematics and learning have impact on their learning goals (Cobb, 1985). Research on students learning at tertiary level has pointed out the connection between students’ conceptions of learning and their approaches to learning. Entwistle and Peterson (2004) defined conception as “an individual’s personal and therefore variable response to a specific idea” (p. 408). Generally students start higher education with different conceptions about learning. The study by Säljö (1979) identifies 5 conceptions of learning: (1) increasing ones knowledge (2) memorizing (3) acquisition of facts, procedures etc. (4) abstraction of meaning and (5) an interpretative process aimed at the understanding of reality. A subsequent study by Marton, Dall’Alba and Beatty (1993) identifies additionally a sixth conception of learning: (6) as a change as a person.

Research into students’ learning conducted by Marton and Säljö (1976) shows that students usually adopt different approaches to learning. The notions of deep and surface approaches to learning were introduced. The main difference between these approaches is students’ goal – whether to understand the subject or to pass the exam with limited effort or engagement.

One of the studies regarding engineering students (Marshall, Summers & Woolnough, 1999) characterizes the qualitative differences and similarities in the conceptions of learning held by engineering students. Five different conceptions of learning given by engineering students are as follows: (1) learning as memorising definitions, equations and procedures (2) learning as applying equations and procedures (3) learning as making sense of physical concepts and procedures (4) learning as seeing phenomena in the world in a new way (5) learning as a change as a person. These conceptions have influence on how students approach the mathematics offered by different courses.

The issue of conceptual and procedural knowledge has high relevance when considering learning and teaching calculus and also in connection to engineering students and mathematics (Randahl & Grevholm, 2010; Randahl, 2012a). The topic of conceptual and procedural knowledge was extensively considered in the mathematics education research. Different definitions and approaches to examine both kinds of knowledge were proposed (Hiebert & Lefevre, 1986; Baroody, 2003; Star, 2005). There is general agreement about the necessity of some connection between concepts and procedures when learning mathematics. Otherwise the students may generate answers without understanding what they are doing. Or they may have a good intuitive feeling for the concept but are not able to solve the problems. When considering the issue of understanding mathematics Skemp (1979) proposed three types of understanding. They are defined as follows:

- Instrumental understanding is the ability to apply an appropriate remembered rule to the solution of a problem without knowing why the rule is working.
- Relational understanding is the ability to deduce specific rules or procedures from more general mathematical relationships.
- Logical understanding is the ability to connect mathematical symbolism and notation with relevant mathematical ideas and to combine these ideas into chains of logical reasoning (p. 45).

The approaches to learning proposed by Marton and Säljö (1976) correspond well with the concepts of conceptual and procedural knowledge (Hiebert & Lefevre, 1986) and Skemp's (1976) relational and instrumental understanding. Students who adopt the deep approach to learning look for a structure of meaning in the new knowledge. They relate new ideas to previous knowledge and experience (Entwistle & Entwistle, 1991). It is similar with the concept of conceptual knowledge (which focuses on relationships between mathematical objects) and with the concept of relational understanding. Students who adopt the surface approach focus mainly on memorizing procedures and facts. They accepted the ideas passively (Entwistle & Entwistle, 1991). It corresponds to the concept of procedural knowledge and with the concept of instrumental understanding.

4.2.2 The cognitive perspective

The cognitive perspective focusses on the individual student and her/his ability to use the textbook in the process of meaningful learning. The notions of previous knowledge, concept image and concept definition are central.

The previous knowledge is important in the learning process; one learns in relation to what one already knows. Ausubel (2000) proposed a learning theory using the concept of meaningful learning. Meaningful learning is defined as a process through which new knowledge is assimilated by connecting it to some existing relevant aspects of the individual pre-existing knowledge structure.

Novak and Gowin (1984) claim:

> To learn meaningfully, individuals must choose to relate new knowledge to relevant concepts and propositions they already know. In rote learning, on the other hand, new knowledge may be acquired simply by verbatim memorization and arbitrarily incorporated into a person's knowledge structure without interacting with what is already there (p. 55).

There has been some focus on the issue of previous knowledge in tertiary level educational research. For example Thompson (1994) was studying students' understanding of the fundamental theorem of calculus and found out that the students' difficulties stemmed from a weak understanding of previously learned concepts such as function and rate of change.

The terms concept definition and concept image were introduced in order to point out the difference between the formal mathematical definition and the learner's ideas about a particular mathematical concept. They provide a useful theoretical perspective in studies about learning and teaching mathematics, especially at higher level, as definitions are more widely used at these levels when the new concepts are introduced.

Tall and Vinner (1981) state:

> The term concept image is used to describe the total cognitive structure that is associated with the concept, which includes all the mental pictures and associated properties and processes. It is built up over the years through experiences of all kinds, changing as the individual meets new stimuli and matures (p.152).

And further:

> We shall regard the concept definition to be a form of words used to specify that concept. It may be learnt by an individual in a rote fashion or more meaningfully learnt and related to a greater or lesser degree to the concept as a whole. It may also be a personal reconstruction by the student of a definition (p. 152).

Previous research has pointed towards a gap that exists between concept image and concept definition, and it has been argued that this can be a problem when students learn mathematics (Juter, 2006; Tall & Vinner, 1981; Vinner, 1991). Students can form their concept image through different examples of the concept. But also the concept definition has to have a role in the concept image developed by the learner. It is expected that the students entering the calculus course have concept images based on earlier experiences and that these interact with the more formal definitions that are presented.

Engineering students need to understand the ideas behind mathematical concepts in order to use the concepts in different contexts. Conceptual understanding is significant in order to use what one already knows in new contexts. They need a rich concept image. According to Tall (1988) "When students meet an old concept in a new context, it is the concept image, with all the implicit assumptions abstracted from earlier contexts, that responds to the task" (p. 3). Work with multiple representations of the concept makes the concept image rich and it helps the learner to build conceptual connections. Kaput (1992, p. 542) points out the importance of using multiple representations of mathematical concepts as follows:

> Complex ideas are seldom adequately represented using a single notation system. Each notation system reveals more clearly than its companion some aspect of the idea while hiding some other aspects. The ability to link different representations helps reveal the different facets of a complex idea explicitly and dynamically.

4.2.3 The didactical perspective

The didactical perspective is used in this study for characterizing the educational setting that creates teaching-learning environments. The didactical perspective is assumed to help to grasp the institutional dimensions of learning and of the process of approaching the textbook as a learning tool. Bishop (1998, p. 43) says: "The institutional context and constraints should be given greater prominence in research". And further "Institutions develop their own rules, history, dynamics and politics, and this need to be recognized and taken account of in the research" (p. 43). Entwistle and Peterson (2004) claim that the teaching-learning environments provided by the educational setting are characterized by the various type of teaching, e-learning and other forms of support provided, assessment criteria, feedback and workload. As stated before, in this study the main focus is on the learner as an individual who constructs her/his own knowledge often in an interaction with others within an institutional environment. The textbook is conceptualized as a learning tool and the learner is approaching the textbook that has been proposed for the course offered by a particular institution. The didactical perspective in this study focusses on the way the textbook is embedded in the institution offering the course for the students with particular focus on the teaching.

The textbook is part of the particular teaching-learning environments provided by a particular education institution, in this case the university college. The college formulates core curricula with subject matter content, learning goals, assessment forms and list of literature. The core curriculum is based on the central study plan with main goals for engineering education, stated by the educational department. It is expected that the future engineers will acquire sufficient mathematical knowledge and skills to enable them to identify, analyze and resolve engineering problems. The textbook is chosen and proposed by a teacher among the college staff. It is assumed that the textbook should support the learner in achieving the goals for the mathematics course.

The didactical approach taken in this study sought to shed light on how the textbook as a curriculum resource is embedded in the teaching practice in order to promote students' learning. The assumption is that the curriculum implementation is highly teacher dependent. The teaching provides different learning opportunities to students in terms of for example the approach the teacher uses and the nature of problems that are proposed to the students. Because of this exploring the teacher's choices of textbook definitions, examples and exercises through the lens of intended learning goals is important.

The process of teaching is complex and teacher's decisions may be influenced by many factors. The teacher's knowledge, beliefs, and identity are among factors that are emphasised in previous research (Remillard, 2005; Ball & Cohen, 1996; Stein, Remillard & Smith, 2007). The teacher's subject matter knowledge and pedagogical content knowledge shape the way of teaching. The pedagogical content knowledge gives special attention to the needs regarding learning and teaching of mathematics (Shulman, 1986; Steinbring, 1998). When considering teaching mathematics different approaches to instruction have been referred to in the literature. Stipek, Givvin, Salmon, & MacGyvers, (2001, p. 214) emphasise the importance of an inquiry-oriented approach as a "shift away from the exclusive use of more traditional textbook-based teaching, in which the teacher is in complete control and the students' only goal is to learn operations to get the right answer". At tertiary level both the teacher-centred approach and student-centred approach have been referred to (Trigwell, Prosser & Taylor, 1994; Trigwell, Prosser & Waterhouse, 1999).

Biggs (1989) pays attention to the connection between learning and teaching and claims: "Good teaching should minimize those factors that lead to surface learning, and maximize those leading to depth learning and achieving" (p. 15). Based on findings from a number of studies, he proposes that teaching strategies that induce deep learning approaches embody four key elements: (1) appropriate motivational context - provision of a safe learning environment - encouragement of interest by involving students in planning and delivery of learning tasks; (2) learner activity - deep learning is associated with active rather than passive learning; (3) interaction with others - communication and collaborations with peers and educators; and (4) learning materials structured and presented in consideration of the knowledge and experience of the learners.

The issue of the teacher's identity is also essential when considering teaching at tertiary level. Tall (1991) emphasises the different perspectives between a teacher at tertiary level being part of a mathematical community and the needs of the students.
He notices:

> …a mature mathematician may consider it helpful to present material to students in a way which highlights the logic of the subject. However, a student without the experience of the teacher may find a formal approach initially difficult, a phenomenon which may be viewed by the teacher as a lack of experience or intellect on the part of the student. This is a comforting viewpoint to take, especially when the teacher is part of a mathematical community who share the mathematical understanding. But it is not realistic in the wider context of the needs of the students. What is essential- for them-is an approach to mathematical knowledge that grows as they grow: a cognitive approach that takes account of the development of their knowledge structure and thinking process (p. 7).

Thus being part of the mathematical community might influence the way of carrying out instruction. A big awareness is necessary since this can create difficulties for the students. The mathematics proposed by the teachers should not only be the evidence of the mathematical knowledge achieved by the mathematical community but has to take into account the needs of the students. The knowledge of how the students make sense of the mathematical ideas and what approaches might increase their understanding is essential in the process of teaching at tertiary level.

4.3 The notion of affordances

The term affordance was first introduced by the ecologist and psychologist J. J. Gibson (1977, 1979). He defined an affordance as a relation between an organism and an object, with the object perceived in relation to the need of the organism. According to Gibson, affordances are not properties of people and objects but rather properties of the ecology of actors and objects. Gibson claimed that every time we see an object we also attach a meaning to it.

The idea of affordances evolved further through the years and different definitions have been proposed. According to Greeno (1994):

> Affordance is a property of whatever the person interacts with, but to be in the category we call affordances, it has to be a property that interacts with a property of the agent in such a way that an activity can be supported (p. 340).

According to Norman (1988) the term affordance refers to “the perceived and actual properties of the thing, primarily those fundamental properties that determine just how the thing could possibly be used” (p. 5). For example a chair affords support and, therefore, affords sitting. Further the close relationship between the actor’s past knowledge and experience and the ways the affordances are perceived, is emphasized. Norman claims: “I believe that affordances result from the mental interpretation of things, based on our past knowledge and experience applied to our perception of the things about us” (p. 219).

The idea of affordances has been adopted within the mathematics education research. In the context of learning the relativity of affordances and abilities is fundamental (Greeno, 1994). The learner has certain goals and he/she brings previous knowledge, skills, ideas and attitudes to the learning situation. Affordances need to be perceived in order to be realised and they provide both opportunities and constraints (Guin & Trouche, 1999; Kennewell, 2001). Constraints are characterised as norms, effects and relations which limit the possibilities of interaction between learner and environments. The opportunities and constraints are not opposites but complementary.

In this study the idea of affordances is used as a supportive one in order to describe and analyse the potential opportunities and constraints in the process of approaching the textbook with the goal to learn mathematics. I look at affordances within the tradition established by Norman (1988) in the concern for user perception as formed by previous knowledge and context. The textbook, the learner and the setting in which the textbook is potentially used are considered as providing both opportunities and constraints in using the textbook as a learning tool. To succeed in the learning process it is necessary to discover and utilize the opportunities, and to discover and defend the constraints. Interaction between the learner and the textbook depends on many factors for example of the affordances of the textbook and the ability of the student. The affordances provided by the textbook give some learning opportunities but the students can be constrained by, for example, poor previous knowledge. Affordances are also embedded in the educational system. Relating to use of technology in the classroom Drijvers (2003) emphasises that generally affordance of the technological tool “not only depends on the affordance of the technological tool, but also on the exploitation of these affordances embedded in the educational context and managed by the teacher” (p. 78). Also Kennewell (2001, p. 106) claims: “The teacher’s role is to orchestrate the supporting features-the visual cues, the prompts, the questions, the instructions, the demonstrations, the collaborations, the tools, the information sources available, and so forth.”

4.4 Considerations about the perspectives and theories adopted in the study

The cognitive view on learning is used as an overall frame in this study. The focus is on the learner as a cognizing subject and the textbook is conceptualized as a cognitive learning tool. The different perspectives are chosen in order to get a holistic picture of the process under investigation. The assumption is that they offer possibility to explore specific aspects of the process. At the same time the problem of coherence arises and has to be considered.

The textbook in the study is conceptualized as a cognitive learning tool that should assist the student in the learning process. Hence, when considering the coherence of the theories and perspectives adopted in this study, the focus will be on their underlying assumptions on the concept of learning. Social constructivism view on learning assumes that knowledge is individually constructed and socially co-constructed during interactions in the learning environments. The meaning constructed by learners depends on their needs, beliefs, and prior knowledge. Within the epistemological perspective on learning the underlying assumption is that the acquisition of knowledge requires the learners to consider the new knowledge and then construct an interpretation of it. The process is based on experiences, beliefs and previous knowledge.

The social constructivist view and the epistemological, cognitive and didactical perspectives influence each other in a mutual way. Regarding the basic assumption on the learning process, both the social constructivism and the epistemological perspective emphasise the issue of personal conceptualisation of knowledge and learning. The epistemological perspective focuses on the epistemological beliefs of the learner. When the cognitive perspective focusses on the cognitive development of the individual student, the cognitive difficulties are inherent to the epistemological nature of the mathematical knowledge. In order to investigate the learning process when approaching the textbook the notions and theories within the area of learning mathematics has been chosen. Previous research has frequently used the notions of concept image and concept definitions when investigating cognitive difficulties that occur when students attempt to learn mathematical concepts (Juter, 2006; Viholainen, 2008a; Viholainen, 2008b).

The notions of procedural and conceptual knowledge are chosen in order to focus on the engineering students' needs. These notions are embedded in the cognitive view on learning. The didactical perspective focusses on how the textbook is embedded in the teaching practice in order to promote learning of the individual learner. The teaching process has an epistemological dimension when considering both the change of knowledge and view on teaching and learning.

Thus, as I interpret the notions and concepts used in my work, they are consistent and coherent and no contradictions or collisions are built into their combined use. In this study the combination of the used theoretical elements and complementary perspectives has been productive and supportive for the interpretations and results.

5 Methodology

I adopt the interpretation of methodology proposed by Wellington (2000) as “the activity or business of choosing, reflecting upon, evaluating and justifying the methods you use” (p. 22). The methodology of the study has to serve and support the aim of the study, which as mentioned is to examine the process of approaching the textbook as a learning tool from epistemological, cognitive and didactical perspective in order to identify and explore possible opportunities and constraints.

In this chapter the research paradigm and the methods are outlined. A section of methodological considerations is also included.

5.1 Research paradigm in which the study is situated

I perceive research as “systematic, critical and self-critical inquiry which aims to contribute to the advancement of knowledge” (Bassey, 1990; p. 35). All research is situated in a paradigm. A paradigm offers ways of seeing and helps to make sense of what we are looking at. According to Kuhn (1970) the paradigm is “the entire constellation of beliefs, values, and techniques, shared by members of a given scientific community” (p. 75). The process of perceiving and making sense of the world is profoundly influenced by our beliefs about the nature of the reality (Bassey, 1999). Also Lincoln and Guba (1985) emphasises the notion of beliefs and define a paradigm as “a systematic set of beliefs together with their accompanying methods” (p. 15). The concept of paradigm relates to the choice of methods of collecting and analysing the data in the research process. Lester (2005) points out that it is important to acknowledge one’s philosophical stance when conducting research. Further he discusses how the philosophical stance influences the process of making claims, drawing conclusions and considering what counts as evidence. There are two different commonly used approaches to educational research: a *positivist* and an *interpretative approach*. While the positivist approach seeks explanation, prediction and control by assuming that we can observe an objective reality, an interpretative approach replaces the notions of explanation, prediction and control with understanding, meaning and action. Interpretive accounts facilitate dialog and communication between interested parties. Reality is indirectly constructed based on individual interpretations. Interpretive researchers have the idea that the perceived reality varies from one person to the other (Pring, 2000). All knowledge is a matter of interpretation, making sense of what is observed. We try to understand what is going on but we do not judge. As researchers we bring with us more or less experience. Something that we see and describe is obvious, something is only our interpretation. The role of the researcher is to “understand, explain and demystify social reality through the eyes of different participants” (Cohen, Manion & Morrison 2007, p. 19).

This study is placed within the interpretative paradigm. I assume that knowledge is personally or socially constructed and it can be understood through qualitative studies of individuals and contexts. Taking into account the social constructivist view of learning the assumption is that each learner constructs his/her own reality so there are multiple interpretations. All learners bring their own unique interpretations of the world and the researcher has to be open to the beliefs, attitudes and values of the participants. The research process aims in seeking meaning in observed actions and insights to people’s perspectives. The process of approaching the textbook is both complex and complicated. There are many factors that might influence the process and there is no certain truth that we can discover. Qualitative research attempts to study naturally occurring phenomena in all their complexity (Bogdan & Biklen, 2003). It is conducted in natural settings, without interventions into the environments. Savenye and Robinson (2004) write about qualitative research: “It typically involves highly detailed rich descriptions of human behaviours and opinions. The perspective is that humans construct their own reality, and an understanding of what they do may be based on why they believe they do it” (p. 1046).

When using a qualitative approach it is important to ensure that the findings are based on critical investigation. In the present case study methodological triangulation is used in order to combine advantages of different methods, reduce their disadvantages and improve validity (Stake, 1995). Methodological triangulation is a technique to collect and analyse the data. It uses “either the same method on different occasions, or different methods on the same object of study” (Cohen et al., 2007, p. 142). In the present study different methods are used on the same object of study.

5.2 The setting of the study

As mentioned in chapter 1 the study was conducted at a University College in Norway in the context of the basic calculus course for first-year engineering students. The calculus part of the course covered topics from differential and integral calculus such as functions, limits, derivative, and applications of derivatives, integration, and differential equations. In this study the focus was on the derivative concept. The concept of derivative is particularly important for engineering students because of its applications in engineering contexts. James (2001) writes:

> Many of the practical situations that engineers have to analyse involve quantities that are varying. Whether it is the temperature of a coolant, the voltage on the transmission line or the torque on the turbine blade, the mathematical tools for performing such analyses are the same. One of the most successful of these is

calculus, which involves two fundamental operations: differentiation and integration (p. 479).

The textbook *Calculus. A complete course* by Adams (2006) was used during the course and because of this was the object of inquiry in this study. The author of the textbook claims: "The text is designed for general calculus courses, especially those for science and engineering students" (p. xv). More about the reasons the teacher gave for choosing this textbook see Randahl (2012). No teacher manual is available on how to use the textbook. The textbook is used around the world and is a popular one in the Nordic countries. In the period before and when the study was conducted, the textbook beside Narvik University College was also used at other institutions, like for example Norwegian University of Science and Technology in Trondheim, University of Bergen and Volda University College.

Approximately 100 students were enrolled in the course. The students had different mathematical backgrounds. The normal minimum prerequisite was the completion of an advanced mathematical course in upper secondary school (for example *Matematikk 2MX*). Candidates of age at least 25 who had five years working experience could apply for the engineering studies using the rule of the 'real competencies'. However, all students, which did not have the required mathematical background, had to take an intensive one-year introduction course in mathematics *Forkurs* (*Preparatory course* – my translation) at the University College. The curriculum for *Forkurs* included i.a. introduction and treatment of functions, limits of functions, differentiation and integration. Because of the different backgrounds the level of students' previous knowledge in mathematics might be varied.

The teacher who was responsible for this course was an experienced lecturer who was respected by the students. According to the students the teacher was always well prepared for the lectures and was helpful when they had any mathematical problems. The students got a detailed syllabus, indicating topics and sections at the beginning of the semester. So they had the possibility to read the relevant text in the textbook in advance. The students took their own lecture notes.

5.3 Research design

A research design provides a framework for the collection and analysis of data. The focus is on answering the research questions. The methods used in the research should fit the nature of the research questions and generate answers to them. The importance of choice of methods is stressed by Pirie (1998): "The choice of the research methods is a very personal decision; although it will be on this choice that the acceptability of the results will largely depend" (p. 21). The present research can be defined as a case study as it concerns a detailed analysis of a single case and aims to answer the questions how things happen and why. According to Yin (1984, p. 23) the case study research method is "an empirical enquiry that investigates a contemporary phenomenon within its real-life context; when the boundaries between phenomenon and context are not clearly evident; and in which multiple sources of evidence are used".

Stake (1994) categorizes three types of case studies as follows:

- Intrinsic case study (undertaken in order to gain a better understanding of this particular case, not because the case is unique or typical but because it is of interest in itself).
- The instrumental case study (used to provide insight into a particular issue or to clarify a hypothesis).
- The collective case study (the study of a number of different cases).

The present research relates to the second category, instrumental case study. The participants were first-year engineering students from one particular institution, the University College. The research process has started with the observed phenomenon of students ignoring the textbook. I intended to understand why it happens. A hypothesis about existence of some factors that possibly influence students' approach to the use of the textbook was formulated and should be clarified.

The research questions in this study are of exploratory nature with some descriptive aspects**.** The overall question is, as mentioned above: What are the opportunities and constraints when first-year engineering students approach the mathematics textbook as a potential learning tool? I assume that by the analysis of the process of approaching the textbook from the epistemological, cognitive and didactical perspectives some possible opportunities and constraining factors can be revealed.

The methods used in this study aim to:

- Explore the process of approaching the textbook by the students with focus on students' difficulties.
- Examine students' previous knowledge in order to relate it to cognitive demands of the textbook.
- Explore students' conception about learning mathematics, their learning goals and about the textbook's potential role in the learning process.
- Examine the textbook as a potential learning tool. Focus on introduction and treatment of the derivative concept; the nature of the definition and purpose of using it, examples and tasks.
- Explore authors' visions for the calculus textbook and their views about own texts. Focus on how these views correspond

to the results of previous research within learning and teaching mathematics.
    - Explore how the textbook is embedded in teaching practice. Focus on the criteria of the choice of the textbook for a specific course and on the position of the textbook in relation to lectures. Explore how the textbook is used by the teacher during the lectures related to the learning goals. Focus on the choice of definitions, examples and exercises.

The following methods have been used:
- Text and content analysis of the textbook (articles 1 and 4).
- Questionnaires (articles 2 and 3).
- Interviews with the teacher and the students (articles 2, 3 and 4).
- Observations of lectures and task solving session sequences (articles 2 and 4).
- Informal talks with the teacher and students (articles 2 and 4).

Additionally the core curriculum at Narvik University College and the Study plan for engineering students were analyzed.

5.4 Methods

In the following the methods used are outlined.

5.4.1 Content text analysis

In this study, content text analysis was used when analyzing three sections of the 'Differentiation' chapter in the calculus textbook. These three sections were chosen because they cover the issue of introduction and early treatment of the derivative concept that was the subject matter of the text analysis. The predominated categories were used. The categories considering the kind of definitions and emphasis on conceptual and procedural knowledge were created on the basis of the theoretical background (Randahl & Grevholm, 2010).
A framework was proposed with focus on:
- The context that was used in order to introduce the concept of derivative.
- The kind of definition (formal or informal) used in introduction of the concept. The notions of formal and informal definition were defined as follows: The formal definition – the concept definition accepted by the mathematical community (Tall & Vinner, 1981), the informal definition – the verbal explanation of the concept without using mathematical symbols.
- Treatment of the global and local aspects of the concept.
- The kind of previous knowledge required in order to make sense of the concept introduced.
- The role of the definition in further treatment of the concept,

proposed examples and exercises. For the categories used see Randahl & Grevholm (2010).

The content text analysis was also used when investigating the calculus textbooks authors' responses to a questionnaire (see section 5.3.2).

5.4.2 Questionnaires

A questionnaire as a method to collect the data was used two times in the study. The first questionnaire was distributed to approximately 90 students in the beginning of the calculus course during one of the lectures. Students needed about 40 minutes to respond. The questionnaire consisted of eight mathematical questions and seven non-mathematical questions. The mathematical questions were placed in the first part of the questionnaire. The questions considering students' mathematical knowledge were of open response format. The questions considering students' believes and attitudes to learning mathematics were of fixed-format. The aim was:

- To obtain insight into students' previous knowledge with focus on ability to make sense of a given definition of the derivative.
- To obtain insight into students' procedural and conceptual knowledge.
- To obtain insight into students' assumed choice of learning sources and their ideas/beliefs about mathematics and learning mathematics.

The design of the questionnaire was guided by the research questions and literature review. The questions were tested with a small group of students from another department in order to get some comments regarding language and clarity. The questions were also discussed with other teachers from the University College and with the supervisors.

As mentioned above a questionnaire was also sent to the authors of calculus textbooks. The aim was to explore authors' views about their texts. The focus was on textbooks' authors' visions about their texts and approaches they choose when new concepts are introduced. The questions were based on the results of the textbook analysis, theories used in this study and research about learning and teaching mathematics.

5.4.3 Observations

Observation as a method was used during the lectures and the task solving sessions. In the beginning (first week) it was unstructured observations with aim to record as much details as possible. The objective was to identify some patterns of students' behaviour and decisions they make in order to obtain knowledge about their approach to learning and using the textbooks. The observations of lectures focussed on the teacher's choices of definitions, examples and tasks. With a *lecture* I refer to a 45 minutes scheduled oral presentation on a preannounced topic to a large group of students (Bergsten, 2007). During the observations the following was of concern:

- To what extent do the students use/not use the textbook? How often do they look into the book? With what purpose?
- How do the students use the book; reading the text, looking at examples, working with exercises others…
- Do the students cooperate with each other?
- What kind of questions do the students ask? And where do they get answers from?
- Do the students ask the teacher for help? What do they ask about?
- What characterizes the help and support from the teacher?
- References to the textbook made by the teacher during the lectures: How often and what kind of references?
- To what extent does the teacher follow the textbook? What definitions, examples and exercises are chosen? Are there any modification of the textbook's definitions, examples and exercises that are observed during the lectures?

5.4.4 Interviews

I view interview as a conversation with a purpose. The students and the teacher were interviewed and I perceived them as key informants. According to Goetz and LeCompte (1984) key informants are "Individuals who possess special knowledge, status or communication skills and who are willing to share that knowledge with the researcher" (p.74).

The semi-structured interview was adopted in this study. A set of questions was prepared based upon data obtained from observations and questionnaires. It was the kind of interviews that Goetz and LeCompte (1984) call confirmation survey. The aim was to get verification of earlier findings mainly from the observations and some informal talks. To establish different perspectives five students with different background and gender were chosen for interviews: two Norwegian female students, a foreign male student and two Norwegian male students. They were chosen because they could provide both insights and detailed information. Because the interviews of two students offered very little relevant data, only the data collected from three interviews was considered in this study.

The aim of the interviews with the students was mainly to investigate students' experiences with using the textbook. The focus was on:

- Personal interest in mathematics and beliefs about mathematics.
- Approach to learning mathematics and learning goals.
- Comparison of earlier and present experiences with using the mathematics textbook, what the differences were.
- Opinions about the currently used textbook.

- Reasons for possible problems with the textbook.

The students were also given a mathematical task selected from the textbook. The aim was to observe how the students use the textbook when they work with textbook tasks.

The interview with the teacher had focus on:

- By whom and how the specific textbook used in the course was chosen.
- The subject content and didactical reasons for choice of the specific chosen textbook.
- Previous experiences with use of the calculus textbooks.
- How the current textbook was intended to be used.

The interviews were tape recorded. The interviews with the students were transcribed in preparation for the analysis.

5.4.5 Informal conversations

According to Cohen et al. (2007) informal conversations are characterized by questions that emerge from “the immediate context and are asked in the natural course of things” (p. 413). Conversations were mainly built on and emerged from lecture- and task solving session’s observations. I usually talked with the students either after lectures and task solving sessions or during breaks. It was quite easy to get answers on questions related to what I observed and it gave me opportunity to get some explanations about students’ behaviour observed during lectures and task solving sessions. Moreover I used to ask the teacher about things that I observed during the lectures regarding her introduction of the concept and choice of examples and exercises.

5.5 Data analysis

The way data is analysed is usually related directly to the research strategy and paradigmatic position. The study was, as mentioned above, conducted within the interpretative paradigm.

I adopted an interpretive approach to data analysing. This approach as outlined by Miles and Huberman (1994) attempts to search for a holistic view of data.

When analysing the data I aimed to relate the data to the theoretical notions that I assumed to be relevant to the study and which are outlined in the theoretical framework. The theory should help me to understand what happened when the students approached the textbook and to focus on the specific aspects and perspectives.

In the first article the deductive approach to data analysis was applied. The data was the mathematical text in the calculus textbook. The aim was to analyse the way of introducing the derivative concept and the kind of examples and tasks used in order to learn about the derivative concept. The analysis was performed by using the proposed framework. The categories concerning the kind of definition, emphasis on conceptual and procedural knowledge were defined with background in the outlined theories. Examples were examined with focus on justification and the new context aspect. Tasks were examined with focus on conceptual and procedural knowledge and possible links between these kinds of knowledge. Although the methods were mainly qualitative, some quantitative steps were used e.g. studies of the number of tasks that emphasises the procedural or conceptual knowledge have been included.

The data in articles 2, 3 and 4 consisted of observation reports, interviews, informal talks and responses to questionnaires. The interviews with the students were transcribed. The process followed the recommendations given by Cohen et al. (2007) and Kvale (1996). The analysis followed the deductive approach since I had some theoretical concepts to observe and explore in the field. The analysis involved the following stages:

- Reading the notes of observations, interviews and informal talks.
- Noting key episodes.
- Identifying topics that were significant for the focus of the study.
- Looking for patterns, links and relationships.
- Looking for factors which are likely to appear together.
- Looking for which factors become visible prior to others.
- Searching for justification for interpretations.

Students' responses to the mathematical part of the questionnaire (article 2) were analyzed with focus on the issue of concept image, concept definition, and procedural and conceptual knowledge in order to relate it to cognitive demands of the textbook.

In article 4 definitions, examples and tasks offered by the teacher were analyzed with focus on the procedural and conceptual knowledge. These were the categories used when examining the examples and tasks proposed in the textbook (Randahl & Grevholm, 2010).

5.6 Methodological considerations

In this section I briefly give an account about the methodological approach adopted in this study in relation to existing methodologies within textbook research. I also make some considerations about the advantages and limitations of the methods used. Some reflections related to the population are also included.

Methodological issues are important in the research on mathematics textbooks (Fan, 2013). During the last years some further substantial developments in methodological approaches within textbook research have been performed. Rezat and Strässer (2012) offer an overview of methodologies used in mathematics textbooks research in Nordic countries. Using a model *textbook-student-teacher-mathematics* in order to structure the field, the overview draws attention to established methodological approaches and methodological deficiencies.

The choice of methodological approach in this study was based on the nature of the phenomenon under study and it is related to the theoretical background. The methodology is embedded in what the study was aimed to explore and understand. The holistic approach to the phenomenon under study requests to look at different aspects of the issue. As stated above, the research was conducted within the interpretative paradigm. Perceiving the process of approaching the textbook as being complex the aim was to characterise the influencing factors through seeking meanings in observed actions. The theories and notions used in this study are of cognitive nature and the adopted methods seek to understand the world of human experience (Cohen et al., 2007).

In the beginning of the study it seemed reasonable to start with an investigation of what the textbook offers for learning. The study in the first article is grounded on the content text analysis. The content text analysis was used to analyse some chapters of the textbook for learning opportunities when offering particular definitions, examples and tasks. The rather narrow selections of text under consideration might be perceived as a limitation. However, the aim was to focus specifically on the introduction and early treatment of the derivative concept. The three chosen sections cover the issue of interest.

The studies presented in the other articles use observations, questionnaires, interviews and informal talks as main methods. These methods have been frequently used in previous research about mathematical texts (van Dormolen, 1986) or about the use of the textbooks when learning mathematics (Lithner, 2003). Considering the teaching I was interested in the choices the teacher made when proposing definitions, exercises and tasks. Because of this aim the observations and informal talks was chosen as useful methods when designing the study.

The reason for choosing several methods was that the research would be strengthened by using several sources of information. There are several potential limitations in using questionnaires. Bryman (2004) mentions some of them: Students may omit key terms in a question, they may vary in their interpretations of the key terms, and they can provide an honest reply. Generally the answers on questionnaires might show only what respondents want us to know. We cannot necessarily be certain that the respondents reply truthfully. The other crucial problem in using questionnaires is that students can interpret the same questions or words differently. The limitations were taken into account by pre-testing the questions with another group of students and discussing the design of the questionnaire with the teacher and the supervisors. Since I was present while students completed the questionnaire they had the opportunity to get answers to any questions. Although the students were positive to participating in the study there were problems with collection of answers to the questionnaires. Only 50 of 90 students responded. One of the advantages of using the questionnaire as a method for data collection was in connection with inquiry about authors' views. It was only possible to get responses from the authors by using questionnaire (Randahl, 2012b).

Many of the responding students did not answer the mathematical questions. Asking them about reasons for this, many answered that they had forgotten the previous mathematics. None of the students gave lack of time as a reason for not giving answers to the questions. I wondered why the students not even tried to give answers to these tasks. One of the reasons might be that they did not have enough knowledge to give a completely correct answer and therefore chose not to answer at all. They could be afraid that by trying to partly answer the tasks they could show some serious misconceptions. Some respondents want to present themselves in favourable terms (Borg & Gall, 1989). So it was obvious that I had to collect more information by using other methods, such as observations, interviews and informal conversations. Bryman (2004) pointed out that use of other methods like observation and interview, can give a more consistent picture of the studied phenomena.

Before conducting observations I considered my role as an observer. Should I be participant or a nonparticipant observer? According to Borg and Gall (1989) "the main advantages of nonparticipant observation are that it is less obtrusive than participant observation and less likely to be distorted by the emotional involvement of the observer". Taking into account the type of population I was observing I decided to conduct nonparticipant observations. More considerations about my role during the task solving sessions are outlined in article 2 (Randahl, 2012a). It was also problematic to get students to allow themselves to be interviewed. I selected ten students on the basis of observations and asked them to participate in interviews. Finally five students agreed to be interviewed. I noticed that the students were generally more reserved when talking about their previous experiences with mathematics and about learning mathematics. They tried to avoid solving the mathematical tasks. There were no such problems with questions regarding the textbook and how they viewed the textbook as a learning tool. Collecting data using different methods was a helpful strategy. Considering the low degree of response on the questionnaire I was concerned that the students might be reluctant to express their actual beliefs and opinions. Because of this I used the method of informal conversations. The informal conversations were a useful source of information and gave me the opportunity to have more contact with the students and get more personal opinions. The students were more eager to tell how they experience the textbook when trying to use it. They referred to their experiences from the lectures and their homework. To avoid getting the information that could be too subjective and less representative of the population, I tried to talk with different groups of students.

When considering observations of the lectures (Randahl, 2012a; Randahl, 2016) a comment has to be made. The teacher was informed that the research would focus on the textbook's role. She told beforehand that she was using her own notes. But my presence could possibly influence her behaviour, for example increase or decrease the number of references to the textbook.

Finally, as mentioned previously, the study was conducted at a tertiary institution with relatively little experience with educational research. The present study should be the first stage in an attempt to focus more on didactic issues within engineering education and possibly create a didactic research community. The study was conducted during a limited period of time and this affected the scope of the study. Furthermore the participants were first-year engineering students and the research was conducted in the time where they tried to be familiar with the new learning-teaching environments.

In the qualitative research the researcher has a central position when the data is collected, analysed and interpreted. This position requires a critical reflection on her/his potential biases as they may influence the conclusions. First of all the comments of my supervisors helped me to think deeply and defend my statements.

6 Results

In this chapter the results of the articles are outlined. The results are based on the data analyses of the textbook, protocols from observations of lectures and task solving sessions, questionnaires with students and textbooks authors, interviews with students and the teacher, curriculum texts and informal talks with the teacher and the students.

6.1 Summary of the results related to the articles

6.1.1 Article 1

The subject of the study presented in the paper was the calculus textbook itself. The aim was to explore the introduction and treatment of the derivative concept. Regarding the introduction of the concept the focus was on the kind of definition, its aspects and role. The other issue of interest when analyzing the textbook was that of procedural and conceptual knowledge. The focus was on the connections between the knowledge and what kind of knowledge was emphasized. The following research questions were posed:

1. What characterizes the introduction of the derivative and the further treatment of the concept in the calculus textbook for first-year engineering students?
2. What kind of knowledge does the textbook emphasise?

The analysis shows that there is no practical context when the derivative is introduced. The presentation of the derivative concept is formal and based on the limit definition. The introduction of the concept does not emphasize the local and global perspectives of the concept.

The treatment of the concept emphasizes procedural knowledge. It is found that 54% of the exercises are placed in the category *Exercises which mainly require the use of particular procedures* and only 20% are in the category *Exercises in which justification of the solution is required or new context is used*.

The other observation is that the set of exercises proposed for the learner is sorted in a particular way. The exercises which require easy procedures for obtaining correct answers are placed in the beginning of the set. There can be a large number of such tasks. Exercises that emphasise conceptual knowledge are placed later in the set. One of the consequences of such a task set structure might be that the time constraints prevent students from focussing on the conceptual tasks.

6.1.2 Article 2

The aim of the study presented in the paper was to investigate how students used a textbook in association with a traditional lecture approach to teaching the concept of derivative. In the paper the emphasis was on how students responded to the contents of sections in a fairly standard textbook (Adams, 2006). The following research questions were posed:

1. What characterises first-year engineering students' approaches to mathematics textbooks?
2. What possible opportunities and constraints might influence the ways the textbook are approached by the students?

The process of students' use of the textbook was explored from epistemological, cognitive and didactical perspective. The focus was on possible opportunities and constraints that might influence the way the textbook was perceived and used by the students.

In the beginning of the semester a questionnaire was distributed. There were 50 students who responded to the questionnaire. Of these students, many did not answer key mathematics questions. The mathematical questions were based on expected students' competencies from upper secondary school. The purpose with the mathematical questions was to explore students' understanding of concepts like *the function, the limit* and *the derivative* that were taught in the secondary school. The assumption was that one understands the idea behind the concept of derivative if one can explain the definition in own words and can make sense of the symbols used. From the answers given, it seemed to be the case that the students entering the course knew very little about important calculus concepts. Only four out of 50 students attempted to make sense of the given definition of the derivative concept. None of the students answered the question regarding using the definition to derive a formulae and justification of the formulae. Relating to the didactical notions of concept image and concept definition, the students showed weak concept images. It gave some indication about possible problems with making sense of the formal definition presented in the textbook.

The majority of the responders (78%) chose understanding as being most important when learning mathematics. The lecture notes and the textbook were pointed out as playing the most important role when learning calculus.

From the interview of the teacher it was found that the department did not have any established procedures for choosing and evaluating the textbook. Choosing the textbook for the course was mainly based on the following criteria: the 'tradition' criterion - the textbook was widely accepted and used in the international mathematics community, and more specific criterion – the textbook was recommended by other teachers from the mathematical community. The informal talks with the teacher indicated that the students were expected to have problems with using the textbook.

The observations of lectures and task solving sessions showed that the students were using the textbook mainly when working with tasks.

When they asked the teacher for help, the questions were about tasks. None of the observed situations indicates that the students sought for explanations connected with the textual part of the textbook. The teacher provided explanations based on students' questions. She did not encourage the students to read the text in order to find explanations.

Students' preference to use lecture notes was confirmed during the observations of lectures and task solving sessions. It was confirmed by the interviews. During the interviews the students talked about the textbook proposed for the course as being huge, including many irrelevant details, and useful mainly as a source of examples.

Both the observations and interviews indicated that students' previous experiences with the mathematical textbook in upper secondary school and their approach to learning influenced how they perceive and use the textbook at tertiary level. The two Norwegian interviewees emphasised that they had become accustomed to reading mathematics textbooks at upper secondary school. According to the students the text was less demanding than in the calculus textbook and the teacher 'guided' the way to use the textbook. In the beginning of the calculus course the students tried to read the textbook but they soon realized that it was too demanding. So they gave up and began to prioritize the lecture notes. The textbook became the source of tasks and examples. The third interviewee, the student from Asia, used the textbook extensively. According to her, working with the text in advance was helpful to understand the mathematical topics that were under consideration during the lectures. She perceived the theory as necessary to understand when studying calculus.

During the interviews and the informal talks the issue 'what mathematics for engineers' was addressed. The two Norwegian students perceived the theory as less important for engineers and they emphasised the importance of working with tasks. This view was also prevalent among students who participated in informal talks.

6.1.3 Article 3

The aim of the study presented in the article was to investigate what the textbook authors think about their texts. The issues of interest were authors' approaches to the learning and the teaching of mathematics and how they perceive the role of the textbook. The assumption was that the authors had to know how students learn mathematics in order to propose meaningful problems, examples and tasks. Three research questions were posed:

1. What characterises authors' vision of the calculus textbook offered to the first year students?
2. What characterises authors' views about the introduction of new mathematical concepts?

3. In what ways do these views correspond with the results of previous research?

The questions in the questionnaire were elaborated based on the theory used in articles 1 and 2 and on the results and findings in these two papers. Four authors responded on the questionnaire. The responses given by the authors indicated that they view teaching in terms of transmission, so they focused mainly on getting the mathematical content 'correct' and 'clear'. The dominant view was that the role of the textbook was to help students to learn by explaining and clarifying. The authors preferred the approach that introduces new concepts based on the traditional way of perceiving mathematics as a system of definitions, examples and exercises. Although all the authors pointed out the importance of understanding calculus, the promotion of such understanding was perceived differently by the authors. One of them related it to "many training problems with full solutions – all calculation steps are shown and in addition even commented". Other authors emphasised the importance of 'conveying' understanding to the learners.

The issue of how the authors' answers are related to the results within educational education research is considered in the conclusion section of the article (Randahl, 2012b).

6.1.4 Article 4

The aim of the paper was to explore how the textbook was used by the teacher during the lectures. The focus was on the choice of definitions, examples and exercises proposed by the teacher when introducing the derivative concept. The following research questions were posed:

1. To what extent does the teacher adopt or follow the textbook's approach to introduction and early treatment of the derivative concept during the observed lectures? What modifications, if any, are done?
2. How does the selection of examples and exercises fit with the learning goals for engineering students?
3. How are the learning opportunities/constraints (as pointed out in the content text analysis) of the textbook utilized/overcome by the teacher during the implementation?

The issue of how the textbook was usually chosen for a particular course was investigated during the interview with the teacher. Results showed that there was no established procedure for choosing the textbook. Students' previous problems with the use of calculus textbooks were not sufficiently analysed and discussed by the teaching staff. The process of choosing the textbook used when the study was conducted was mainly based on opinions and hints given by colleagues from other institutions.

The other issue of focus during the interview and informal talks with the teacher was that of students' previous problems with a calculus textbook. Taking into account students' complaints, the teacher seemed to have low expectation about the use of the textbook by the average student. The teacher was aware that the lectures were perceived by the students as important and that the students used lecture notes frequently. From what the teacher said during informal talks it was obvious that the teacher agreed with students giving high priority to following the lectures and to use lecture notes. Some statements showed that the textbook was perceived by the teacher as offering additional examples mostly to the students who were especially interested in mathematics. The position of the textbook was not clearly enough defined in the calculus course, the implicit message send by the teacher emphasised lecture notes.

The informal talks with the teacher indicated that the textbook was an important resource for the teacher when preparing and deciding the content of the lectures. It was confirmed by observations of the lectures. The textbook's approach was closely followed when the concept of derivative was introduced. The sequence of formal definitions from the textbook was presented. When introducing the concepts the teacher tried to give explanations as clearly as possible. No problems were posed in advance of the definitions and examples.

The majority of the examples and exercises were of procedural character. The choice of examples and tasks did not utilise all opportunities the textbook provided. Only a few examples and exercises promoting conceptual knowledge were discussed during the lectures. Some exercises that potentially might extend understanding of the concept (as shown in the content text analysis) were omitted by the teacher. The possibility of focussing on 'switching' between different representations in order to extend students' understanding of the concept was not utilized (Amoah & Laridon, 2004). The choices of the examples and tasks indicate the teacher's beliefs that the procedural knowledge was most important for the students.

7 Discussion

In this chapter the main results from the articles are discussed in relation to the aim of the study.

7.1 The process of approaching the textbook as a learning tool

Based on the observations during the pilot study the following hypothesis was formulated before the main study: There are possible factors that influence first-year engineering students' approach to the mathematics textbook as a potential learning tool. Thus the study intended to explore the process of approaching the textbook from different perspectives in order to identify these factors that might support and constrain the process of using the textbook.

The research in this thesis was guided by the following research question: What are the opportunities and constraints when first-year engineering students approach the mathematics textbook as a potential learning tool?

The main study confirmed the observed phenomenon from the pilot study. The observations, questionnaire and interviews confirmed students' intentions to use the textbook when entering the calculus course (Randahl, 2012a). However, a decreasing interest for using the textbook by the students was observed during the observation period. The obtained results indicated different factors of epistemological, cognitive and didactical character that might influence students' perception and initial use of the textbook. In the following the discussion of the factors are structured according to the three perspectives.

7.1.1 Epistemological factors that support and constrain students' use of the textbook

In this section the epistemological factors related to the learner are discussed. The epistemological factors that refer to teacher's beliefs and views of the nature of mathematics knowledge that is needed by engineering students and teacher's beliefs about teaching and learning are discussed in section 7.1.3.

The results of the study indicate that some epistemological factors influenced students' approach to the mathematics textbook as a learning tool. Students' perception of the textbook was clearly influenced by the nature of the mathematical knowledge as it appears in the calculus textbook together with their personal views of both mathematics and learning of mathematics.

The idea about the introduction of mathematics concepts at tertiary level is embedded in the axiomatic structure of mathematics (Lakatos, 1976). This structure is adopted by many textbook authors. It has the consequence that nearly all calculus textbooks follow the pattern: definition, examples and tasks when introducing and treating the concepts. The formal approach to mathematics in the textbook was expressed by the students as a clear constraint when they initially evaluated the textbook as a possible learning tool. Some of the students (as they told during the interviews and informal conversations) tried to read the textbook, but they gave up because they perceived the textbook as too difficult. The findings suggested that when the text was perceived as too demanding, the students ignored the book and preferred to use the lecture notes.

Moreover, the students did not perceive the mathematics offered by the textbook as relevant for the engineering context. The observations of how students use the textbook gave the evidence that they mainly skipped the exposition and focused instead on worked examples. The interviews and informal talks with the students confirmed that the formal definition was viewed as not so relevant. They believed that the procedural fluency was essential when considering the role of mathematics for engineers. Because of this they were not interested to spend time on studying the theoretical parts of the textbook. It fits with warnings of Gnedenko and Khalil (1979, p. 73): “Mathematics must not assume the role of an absolute logical system, but, first and foremost, it must be an instrument of learning, a means for the solution of engineering problems”.

Answering the questionnaire, 78% of the students declared that understanding was important when learning calculus (Randahl, 2012a). However, the observations indicated that when the students had problems with solving the tasks in the book they seemed to take a surface approach to learning (Marton & Säljö, 1976; Säljö, 1979). They gave up to read the text in advance and clearly focused on examples in the textbook in order to figure out procedures or they waited for help from the teacher. The surface approach to learning was a constraint when considering the process of meaningful use of the textbook.

One student participating in this study (the student from Asia) showed an entirely different approach to the use of the textbook than the other students. She told that she was reading the text in advance and tried to understand the topic in order to be prepared for the lectures. She focused on individual study of the text in advance. She perceived the text in the textbook as very significant in the learning process. The lectures were perceived by her as giving possibility to ask the teacher if something from the textbook was unclear for her. This indicates that students’ beliefs and attitudes based on previous experiences may shape their way to approach the textbook as a learning tool.

7.1.2 Cognitive factors that support and constrain students' use of the textbook

The study identifies students' poor previous knowledge and cognitive demands of the textbook as a source of students' difficulties. The connection between the cognitive demands of textbook and students' prior knowledge is emphasized in a study of Sosniak and Perlman (1990). Data obtained from the mathematical part of the questionnaire showed that the majority of the students did not seem to have any idea of what the statement defining the derivative of the function means.

The epistemological change of the nature of mathematical knowledge from lower to higher educational level is pointed out in previous research as a possible source of the problems for students who have not developed rich concept images (Vinner, 1991; Raman, 2004; Juter, 2006). For example Juter (2006) when studying students' problems with limits, claims that textbooks used at upper secondary schools do not provide much theory or many tasks in that area, and thus most students do not have a developed image about the limit concept.

By drawing on the relativity of affordances (Greeno, 1994) of the textbook and abilities of the learner, the existence of possible cognitive constraints can be better understood. The introduction of the derivative by use of a formal definition is a cognitive constraint for students with poor concept image of the concept. A poor concept image of the basic mathematical concepts is also a cognitive constraint since it makes it difficult to grasp the meaning of the formal definition. Making sense of the formal definition strongly depends on students' previous knowledge, as a rich concept image is essential. Taking into account the results from text content analysis (Randahl & Grevholm, 2010) of the textbook proposed for the course it is not surprising that the students perceived the textbook as difficult.

7.1.3 Didactical factors that support and constrain students' use of the textbook

When considering the didactical factors that might influence the role of the textbook as a learning tool the issue of how the textbook was embedded in the context of lectures and task solving sessions was of main interest. The assumption was that the format of lectures together with the task solving sessions might provide learning opportunities with meaningful use of the textbook.

The focus was on to what extent the textbook was used by the teacher regarding both the content and the way of introducing concepts. Teacher's choices of definitions, examples and exercises were explored in terms of intended learning goals for first-year engineering students. Additionally the way the textbook usually was chosen for the course was examined.

The observations indicate that both the way the textbook was embedded in the lectures and task solving formats and the approach to teaching might influence students' perception and approach to the use of textbook. The students were not encouraged to read the text in the book in advance to the lectures. As the interviews and informal talks indicate, the students tried to read the text on their own but because of the cognitive difficulties they gave up. Asking some questions related to the text in advance of the lectures could help students to make sense of the formal mathematics. The observed traditional lecture-style did not provide opportunities for the students to actively use the textbook (Randahl, 2016). The teacher adopted a teacher-centred approach when introducing the concept (Trigwell, Prosser & Taylor, 1994; Trigwell, Prosser & Waterhouse, 1999).

The textbook was used by the teacher in a rather reproductive way. Following closely the subject matter from the textbook the teacher did not explicitly make references to the text. The concept of derivative was presented as an existing body of knowledge. When introducing the concept by definition, there was no attempt to show the role of the formal definition. The teacher did not create any environments which by using context, examples and tasks from the textbook explored mathematical ideas and in this way fostered learning. There was not any attempt to modify the tasks from the textbook in order to embed the idea of derivative in engineering contexts. Some problems embedded in engineering context were collected from another calculus textbook.

The teacher's beliefs (as expressed during the informal talks) about what mathematics was important for engineering students were illuminated by the choices of many examples and tasks promoting procedural knowledge (Randahl, 2016). It fits with the claim of Thompson (1992) that teachers' beliefs influence their practice.

The analysis of the textbook (Randahl & Grevholm, 2010) pointed out that the textbook provided many conceptual tasks that could support students' understanding of the derivative concept. At the same time it was noticed that some assistance of the teacher was desirable when the students should work with the textbook. Although the students during the task solving sessions worked with the goal to achieve understanding, they worked in ineffective ways and did not manage to consider more demanding tasks (Randahl, 2012a). Some assistance and guidance from the teacher regarding the choice of tasks from the textbooks could be helpful for learning situations. In the context of task solving sessions students' questions and teacher's responses were of special interest. Although the tasks for the sessions were selected from the textbook, the students were not encouraged to read the text when asking for help. Due to time constraints the teacher often only briefly explained the procedure to the students in order to help them to solve the tasks (Randahl, 2012a). The way of providing help by the teacher (often because of the time limitation) seemed to be a constraint when considering the role of the textbook as a learning tool.

The teacher aimed to help students to achieve an understanding of the concept. But when she did not refer explicitly to the text, the students might easily ignore the textbook. The students' signalized the view that the lecture notes were most important and this fact might be a constraint when perceiving the role of the textbook. According to the students the lecture notes provided learning opportunities: the concepts were explained in less formal ways and many examples were given.

7.2 Sometimes the opportunities became constraints

Basically the textbook should play an essential role in learning mathematics at tertiary level. The availability of the textbook is an important feature that might enhance learning opportunities. The textbook can be used inside and outside the classroom. It means that the students had also the opportunity to read the texts in case they either were not present at the lectures or did not understand what was taught.

The textbook can potentially afford interaction if the learner perceives that the textbook might be useful when learning mathematics (Norman, 1988). It means that the way students use the textbook relates to their perception of it. The perception is further related to for example students' previous knowledge, their learning goals and approach to learning, and also previous experiences with using the textbook. In this study the student from Asia based her approach to the use of the textbook on previous experiences (Randahl, 2012a).

The observations of how the textbook was embedded in the institutional setting showed some interplay between the role of the textbook and the role of lecture notes. It seemed that the way of perceiving and using the textbook by the students was 'affected' by how the students viewed the role of lectures and lecture notes. The observations of lectures and the informal talks with the students gave evidence that students' beliefs about the role of the textbook and their behaviour were changed during the observed period of time. The textbook from being assumed as a potential learning tool became perceived as less useful when learning mathematics.

The study shows that some of the features of the educational setting gave learning opportunities but became constraints when regarding the process of approaching the textbook as a learning tool. During the lectures and task-solving sessions the teacher could explain clearly the definitions and clarify difficult tasks. The students had opportunity to ask questions. But at the same time, if the textbook was not clearly embedded in the lecture context, the lecture format might be a constraint for the meaningful use of the textbook. Perceiving the lectures as offering the clear explanations of the theoretical parts from the textbooks the students rarely cared about own theory study. It might be one of the reasons to their perception of the textbook mainly as a source of tasks.

The issue of how the textbook was embedded in teaching practice was investigated and reported in Randahl (2016). This aspect was studied mainly during the observations of lectures when the teacher introduced the new concept, and of sequences when the teacher provided help to students working with the tasks. When preparing the lectures, the teacher took into account that the formal presentation of the concept could be a constraint when students read the text. They were not encouraged to study the text in advance but the teacher prepared a presentation of the textbook's definition with detailed explanation of every term. This approach to teaching offered much support in making sense of the symbols but promoted students' dependence of following the lectures passively. It also strengthened the role of lecture notes in relation to the textbook's role which was confirmed during the interviews and informal talks with the students. They preferred to read the lecture notes when they needed the theoretical part of calculus. The well-developed explanations during the lectures were perceived by the students as learning opportunities. But these explanations without specific references to the text were clear constraints when considering students' active use of the book.

The teaching of mathematics requires both a solid subject matter knowledge and pedagogical content knowledge (Harel, 1993). By being aware of students' possible problems with the use of the textbook in a learning process and by defining the role of the textbook the teacher can help students to use the textbook in a meaningful way (Wagner, Speer & Rossa; 2007). Some constraints provided in the teaching sequence in form of inquiry questions referring to the text might change the role of the textbook and influence the nature of learning.

The textbook proposes many tasks that emphasize procedural knowledge giving engineering students the possibility to learn procedural skills. The book also proposed plenty of tasks that emphasize conceptual knowledge in order to understand the ideas behind the concepts. Working with the tasks provides an opportunity to achieve such knowledge. But these tasks are usually placed at the end of the exercise section in the textbook. Thus if the textbook is not meaningfully embedded in the course and the students are not encouraged to work with these tasks they might never work with them because of for example time constraints (Randahl, 2012a). In this way the big amount of procedural tasks placed at the beginning of the task section in the textbook might be perceived as a learning constraint. When working solely with them the students might believe that they develop understanding of the concepts.

The explicitly defined goals of the core curriculum might be regarded as possible opportunities when approaching the textbook. Students might define their own learning goals in the calculus course and relate them to the textbook. The issue of perceived and recognized affordances is emphasised by Greeno (1994). Although the students might first perceive the textbook as difficult to use, the relevant connection between the mathematics taught during the lectures and the mathematics presented in the textbook could help them to recognize the learning opportunities of the textbook. But when the goals of the curriculum are not clearly related to use of the textbook and the textbook is not referred to during the lectures, the students can easily ignore the book and perceive the lectures as more important and useful than the textbook.

As the lecturer stated in the quotation (Randahl, 2012a, p. 249) the textbook was huge and included many things that did not appear to be of concern to the students. Such a statement implies the question - if that was indeed the case, then why did the lecturers prescribe the book as the set text? The issue of evaluating and choosing the textbook for a specific course should be emphasised and clearly embedded in the teaching practice at the tertiary level.

The answers given by the authors of the textbooks to the questionnaire (Randahl, 2012b) confirm concerns whether the curriculum materials are based on research into students’ learning and understanding of the concepts. It seems that the textbook authors do not adjust well enough to the previous knowledge and interests of the audience they are addressing, and in turn making assumptions that are totally unjustified and rarely giving sufficiently simple explanations. As a result the students' decision to seek other ways of learning (like for example using lecture notes) seems entirely rational.

8 The quality of the study

8.1 The quality of the study according to the criteria for scientific quality

The quality of this research is discussed according to the criteria proposed by Lester and Lambdin (1998). These are: worthwhileness, coherence, competence, openness, ethics, credibility and clarity.

According to the criterion of worthwhileness, the study should be considered as a solid body of knowledge and it should add to and deepen our understanding of issues within learning and teaching mathematics. Since the area of mathematical education at tertiary level is still young, there is a need for knowledge that is empirically based (Speer, Smith & Horvath; 2010). The present study offers empirical evidence for the issue of interest. The study is clearly situated in the existing body of knowledge and expands the area of tertiary level research. The previous textbook research was to a high degree characterized by research on content and comparative research and focused on lower educational levels. By using different perspectives, including the didactical perspective focusing on the educational setting, the study points out the complexity of the issue and gives a holistic picture of what factors may influence the textbook's role at tertiary level.

To my knowledge there are no existing studies which consider the textbook authors' views about own texts and the approach to learning and teaching mathematics at tertiary level. The inquiry into authors' thoughts and visions enlightens and extends our knowledge about which ideas are embedded in the textbook proposed for the calculus course. It gives the study the potential for informing the mathematics education practice at tertiary level.

Coherence is concerned with the correspondence between the research questions and methods used in this study. When particular methods are suggested, it is important to justify them. Lester (2005) claims that it includes "clarifying and justifying why a particular question is proposed to be studied in a particular way and why certain factors (concepts, behaviours, attitudes, societal forces) are more important than others" (p. 460). The research questions in the articles are posed in order to enlighten the observed phenomenon. The questions seek explanations which demand an 'in-depth approach'. This is achieved by using qualitative methods and triangulation.

Competence of the researcher is crucial. The way of sample selection, data reduction, interview design and choice of appropriate data analysis tools depends on the proficiency/skills of the researcher. The study was conducted in the context of a calculus course. All students participating in the lectures and task solving sessions were participants of the study. Students with different ways of using the textbook (conclusions from observations and informal talks) were selected for interviews. The way of selecting and analysing the data are described in detail and transparently in the articles.

There are two aspects of the criterion openness. The first is that the assumptions for the study should be clarified and how the data was collected and analyzed should be explained explicitly. The second one is that the used methods should be described well enough to make it possible to investigate them. I mainly met these demands through explicit considerations of assumptions and methods used in each article included.

The criterion of credibility is about justification of the results, whether we can trust the results. The claims and the conclusions drawn in the study should be justified. All the results in the study are supported by the data. In every article included, the results are justified.

In writing up the study I have strived to achieve as much clarity as possible. A number of colleagues (and my supervisors) have read my entire texts and assisted me in finding better formulations, where the clarity was not good enough. I strived for well-structured sentences with clear meaning and formulations and the use of common words. Although writing is not my strongest competence, I have worked hard for clarity and hope that with the help of many other persons the text is now transparent and easy to follow. As neither English nor Norwegian is my first language, there may still be expressions that are not typical for the English language. The text has been checked and improved by a native English speaking person.

The research study is also a process of making moral decisions. Bryman (2004) points out that the role of values in the research process becomes a topic of concern. To face difficult situations and judge what is right, what is necessary and what is obligatory, is not always easy. The ethical considerations can also influence the way in which one reports the results of the study.

There were clear principles which I followed during the study:

- The students were asked to participate voluntarily in this study.
- The identities of students are protected.
- The authors of the calculus textbooks were assured anonymity.
- The data are presented in ways which make it impossible to identify the participants.
- My presence as observer in the group setting/lectures should not be perceived as uncomfortable for the participants.
- The respect for the participants was quite obvious in all research situations, perhaps especially when a student asked for help.

8.2 Reflections and critical considerations about the study

The main study started in 2007. The first article was finished in 2009 and published in 2010. The fourth article has been submitted to an educational journal in 2014. Working with the overview during recent months gave me opportunities to look back and make some reflections. The methodological considerations are offered in section 5.5. In the following I attempt to look critically both at the articles and at the whole study.

The main idea behind the study was to explore the textbook's role in an engineering setting. The focus of the investigation was subdivided into the opportunities and constraints that might be imposed by different factors enlightened by the epistemological, cognitive and didactical perspective.

The study was conducted from a cognitive orientation. The textbook was conceptualized as a learning tool, a cognitive tool that should promote cognitive processes related to meaningful learning. I looked at the process of approaching the textbook from epistemological (the mathematics proposed in the book, students' view about mathematics and learning), cognitive (students' previous knowledge, cognitive demands of the formal mathematics) and institutional (how the textbook was embedded in a particular course) perspectives. Using the different perspectives gave some opportunity to get a more holistic view on the issue of investigation.

The first results showed that the textbook promotes formal mathematics and the students had weak previous knowledge. Consequently they ignored the textbook, it was too difficult for them to read the text; they used the textbook mainly as a source of tasks.

Taking the didactical perspective I intended to explore how the textbook was used by the teacher. Lecture observations showed that the teacher followed the textbook quite closely but there were only infrequent references to the textbook. The attempt to analyze the findings in term of teacher knowledge in relation to the use of the textbook forced me to look for relevant literature and I found it stimulating. I became more aware of the importance to consider the role of the textbook within teaching practice at tertiary level.

However, when I look back, the cognitive orientation did not help me to understand and explain the data fully. Reading some texts of Michael Apple (2010) turned my attention on the social and political dimensions of the educational contexts. According to Apple the distribution and use of the textbooks express the social, political and economic relations to the wider context in which they are situated and established. To analyze the data when taking into account the context of the study could extend understanding why the textbook was used in the way as observed. Some factors like the conditions that teachers face (for example time constraints: the teacher could only give short answers when the students had problems during the task solving sessions), the strength of the state, the realities of students' lives, and the political economy of text publishing could be significant. I sent a questionnaire to the authors of the most used calculus textbooks. Some answers indicated that the authors were only allowed to make small changes in later issues of their books. Additionally there are only a few books in the market and I think it is quite difficult to propose a new one by some young authors.

These considerations indicated that the issue of how the textbook is used and embedded in the context of a course proposed by a specific educational institution expresses some social, political and economic relations to the wider context (for example state, publishing house) in which they are situated. These are worthy to be paid more attention to in the future research.

The view on the use of the textbook by the Asian student draws attention to the possible role the culture plays when considering the way students approach the textbook. I realize that I could focus more on this student when collecting the data. However, the culture aspect might be considered to a greater extent in the future research.

When I looked back at the articles, I see that some key concepts and issues could be better defined and emphasized. The analysis of the textbook (Randahl & Grevholm, 2010) was an important work that made foundations for the rest of the study. However, some of the key concepts like for example *learning* and the *textbook as a learning tool* could be conceptualized more explicitly in the paper. Also the issue of the engineering students' needs when learning mathematics should be emphasised early in the study. In the kappa I attempted to describe the concepts more specifically and deepen the relevant issues.

When exploring the literature about the textbook's role in learning and teaching the absence of the voice of textbooks' authors was noticeable. So taking into account authors ideas and views about their texts was important when considering the role of the textbook (Randahl, 2012b). The responses to the questionnaire showed that these authors mostly have an idea of teaching as transmission, so they focus mainly on getting the content correct and "clear." Reflecting on the questions posed in the questionnaire I realize that they were mostly about what the authors thought about their texts, not much about what they understand about learning. Because of this it might be hard to know much about their visions of learning from their answers. Focusing more specifically on the learning, when posing the questions, could force the authors to reflect on their products as tools of learning.

The aim of the study was not to propose any model for the use of the textbook. It was rather important to enlighten the process of approaching the textbook in an attempt to identify the factors that could be useful in future considerations of a possible model for implementation of the textbook in the learning of mathematics at tertiary level. In this matter the model called a didactical tetrahedron proposed by Rezat and Strässer (2012) has to be taken into account.

Generally the research is limited in that it provides data from a relatively small sample of participants. Only 50 students responded on the questionnaire and many did not answer the mathematical questions. In qualitative case studies the question of generalization often arises. The conclusions can be made only about the students that participated actively in this study. But the idea behind the study was to explore the mathematics textbook's role in a specific setting. As Savenye and Robinson (2004) claim: "In qualitative research, it is not necessarily assumed that the findings of one study may be generalized easily to other settings. There is a concern for the uniqueness of a particular setting and participants" (p. 1046). However some generalization of results obtained in this study can be done in future research. From the observations and interviews a hypothesis has been stated that there are some factors causing first-year engineering students' decreasing perception and use of the textbook as a learning tool. This study offers some results that may be a starting point for future research in an attempt to make the results more general.

The research is limited to the derivative concept. The concept was chosen because of its significance in engineering context (see 5.2). Future studies can investigate other areas of the content than the derivative.

The results of the study indicate that some factors as for example the approach to introducing of derivative concept, students' weak previous knowledge, students' approach to learning and the way the textbook was embedded in the learning-teaching environments might influence students' perception and initial use of the textbook. Some connections between the factors and way of approaching the textbook came to be visible. Looking back at the study I realized that I could conduct further quantitative inquiry in order to search for correlations or causal connections. Some voices have argued for expanding research issues from descriptive to a more causal focus on the textbook's role in the educational context (Fan, 2013). Also this aspect should be considered in future research.

9 Concluding discussion

9.1 Significance of the research

As quoted in article 2 (Randahl, 2012a) one of the aims of research at tertiary level posed by Artigue (2001, p. 207) is to “improve the understanding of students’ difficulties and the dysfunction of the educational system”. Selden and Selden (2001) identified four major topics of interest in undergraduate mathematics education: the role of technology, the transition from secondary to tertiary education, the need to produce future mathematics teachers and the potential impact of research on teaching and learning at tertiary level.

How does the study contribute to advance the knowledge within mathematics education? What are the new results for the area of mathematics education at tertiary level?

The results of the study support previous research results about students’ poor knowledge about the derivative (Orton, 1983; Amoah & Laridon, 2004). The students in the study focused on the examples when working with the tasks from the textbook, what was similar to the observations by Lithner (2003, 2004).

The results of the study offer some contributions to a better understanding of the interplay between epistemological knowledge, epistemological beliefs, cognitive difficulties, teaching practice and the role of the textbook from the engineering education stance.

There is not much research regarding learning mathematics of students taking mathematics as a service subject. The way a specific group of students at tertiary level perceive and use the mathematics textbook has not been investigated in Norway before. This study provides empirically based knowledge about this issue.

Analyses of mathematics textbooks with specific focus on how central concepts are introduced by definitions, connected to the mathematical theory and illustrated through examples, exercises and problems for the students to work with, is a field of growing interest in mathematics education research. The study expands the analysis perspective on the textbook by focusing on the tertiary level setting. The students at tertiary level may be expected to work with the textbook on their own. Because of this it is important to have insight into what an established calculus textbook offers for the first-year students in order to learn mathematics.

The textbook defines the mathematics and what it means to learn mathematics and it is therefore important to get insights into textbooks authors’ views about their texts. One of the results in this thesis regards textbook authors’ thoughts about their texts. The idea that emphasizes explanation and clarity of presentation of the mathematical knowledge may contrast with the idea of an active learner. The study reports views of only four authors but their textbooks are used by some university colleges and university departments in Norway and because of this influence what mathematics is taught at tertiary level.

Several researchers have pointed out that the students entering the tertiary level have poor concept images of the basic concepts.

The results of the study confirm previous research results about students' problems with the derivative concept (Orton, 1983; Hähkiöniemi, 2006; Zandieh, 1997), here in first-year engineering students' course context. Students entering the tertiary level have poor concept images about the mathematical concepts and have problems seeing the meaning of formal definitions (Hähkiöniemi, 2006). Then the study relates these results to the area of textbooks showing that the formal presentation of the derivative concept creates a cognitive gap and causes students' difficulties with meaningful use of the book.

The results of Lithner (2004) indicate that students perceive the textbook mainly as a source of tasks and they read examples in order to copy them when solving the tasks. The present study confirms these results and additionally uses different perspectives in order to enlighten and get better understanding of students' behaviour. The focus is on identifying and exploring the reasons for students' behaviour from different perspectives. In this way the study offers a broader view on what might influence the role of the textbook as a learning tool.

While previous research focused mainly on students' ways of using the textbook at different levels (Pepin & Haggarty, 2001; Lithner, 2003) this study attempts to explore the process of approaching the textbooks from different perspectives and to identify possible factors that influence the way the textbook is perceived and used by the students.

By pointing out some aspects of the teaching practice at tertiary level that might constrain students' active use of the textbook the study also contributes to a dialogue on how mathematics textbooks can be meaningfully implemented as a learning tool in cooperation with the format of lectures and task solving sessions.

The study also considers the process of choosing the textbook for a particular course. Over the years the calculus textbooks have become more comprehensive, many of them have included CD-ROMs, websites and so on. In this context the main issue of concern for the teacher might be 'what textbook shall I choose for a particular course'. The question 'how can I as a teacher at tertiary level use the textbook' might not be of major concern. The study investigated how teacher practice might contribute to define and strengthen the role of the textbook as a learning tool.

One of the reviewers of the article (Randahl, 2012a) wrote: "The study could be useful as a warning to those involved in tertiary mathematics programs just how crude and deficient standard approaches to the teaching and learning of calculus can be. Despite numerous large reform efforts having taken place, not much appears to have changed over the past 50 years".

9.2 Implications for practice

Identification of factors that might influence the process of approaching the textbook in a learning-teaching process is important in order to extend our knowledge and inform the practice field. Artigue (2001) says: "…, I am convinced that existing research can greatly help us today, if we make its results accessible to a large audience, and make the necessary efforts to better link research and practice" (p. 207).

However, when posing the goals for research at tertiary level Artigue warns: "Finally, research-based knowledge is not easily transformed into effective educational policies" (p. 207).

One of the suggestions supported by the investigations in this thesis is that the teachers should be more aware when choosing the textbook for a particular course. The questions that arose from the study and which have to be considered when choosing the textbook might be as following: *Can the textbook as it presents the content matter knowledge with the approach used be proposed to the actual group of students?* and *What constitutes the textbook as a learning tool*? *How should the textbook be embedded in the learning-teaching environments?*

The study suggests that the process of using the textbook with the intent to learn mathematics may not be straightforward. The process of approaching the textbook when students make the first evaluations of the textbook is crucial for the further role of the textbook in the learning process. In order to establish the textbook as a learning tool it is essential that the textbook is embedded in the course setting.

9.3 Future research

The data of this study may be used to derive questions for later investigation. Some possible questions have been stated in the articles. For example 'How does the implementation of the textbook change the teaching and learning at tertiary level? (See Randahl, 2012a).

The results and experiences obtained in the study could be used in the bigger study that might involve several educational institutions having engineering program. There is a need to know much more about how tertiary mathematics is taught to students enrolled in mathematics service courses.

The issue of ‘relationship’ between the teaching format at tertiary level and mathematics textbook’s role has to be further explored. Using the term of *didactical contract* from the French didactic could be helpful in such an inquiry. Another option is to consider if using the theory of *Three worlds of mathematics* (Tall, 2013) could give more insight in the path of the learning process taken by engineering students when studying calculus. Considering the mathematics for engineers David Tall (private correspondence, 2010) claims that calculus is based on how human beings make sense of things. The combination of embodiment and symbolism might be essential to the engineers. When introducing the concept of the derivative, using the limit definition is not appropriate for engineering students. The engineers will have to handle real world situations by creating the embodied imagery of the situation and translate it into symbolic form to achieve a practical solution of the problem. Such other approaches to the introduction of calculus concepts have to be considered.

As mentioned in the session *Reflections and critical considerations about the study* some cultural, social and political relations might influence the role of the textbook in learning-teaching environments created by institutional setting. These aspects are worthy to be paid more attention to in the future research.

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Appendices

Appendix A

Appendix A contains the four papers:

1. Randahl, M. & Grevholm, B. (2010). Learning opportunities offered by a classical calculus textbook. *NOMAD*, *15*(2), 5-27.
2. Randahl, M. (2012a). First-year engineering students' use of their mathematics textbook – opportunities and constraints. *Mathematics Education Research Journal*, *24*(3), 239-256.
3. Randahl, M. (2012b). Approach to mathematics in textbooks at tertiary level – exploring authors' views about their texts. *International Journal of Mathematical Education in Science and Technology*, *43*(7), 881-896.
4. Randahl, M. (2016, accepted for publication in *International Journal of Mathematical Education in Science and Technology*). The mathematics textbook at tertiary level as a curriculum material - exploring the teacher's decision-making process.

Appendix B

Appendix B contains the questionnaire to students in Norwegian and with a translation into English.

Appendix C

Appendix C contains the questionnaire to the authors.

Appendix A

Article 1

Learning opportunities offered by a classical calculus textbook

MIRA RANDAHL AND BARBRO GREVHOLM

In this paper we present results of an analysis of what the textbook used by the first year engineering students offers the students, when they take a basic calculus course. The aim of this analysis is to examine as an entirety what students are offered by the book to learn about the concept of derivative. The results show that the presentation of the concept is formal and depends on students' previous knowledge. The treatment of the concept emphasises procedural knowledge. It is not easy for students using the book to make connections between conceptual and procedural knowledge of the concept of derivative.

A group of mathematics education researchers (including the authors) are working at a university college in northern Norway. The university college is rather young, founded in 1994 and it has about 1300 students. One of the main education programmes is engineering, with about 150 new students each year. Some years ago a quality reform was undertaken at Norwegian universities with the intentions to improve quality of all higher education (Kvalitetsreformen, 2003). A closer follow up of students and outcomes was demanded. This has resulted in raised awareness among faculty members about issues related to mathematics teaching and learning. Teachers are asked to work for improved recruitment of students, improved contact with students during courses, improved success rate and fulfilling of studies, and improved learning outcomes. Thus when this university college got an opportunity to hire a research student for doctoral studies in mathematics education, there was a wish to carry out a study that could result in better insights into the

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mathematics components of the engineering education. The area of study was chosen to be engineering students' use of the textbook in a first year calculus course. The main aim was to find out what characterises the students' use of the textbook. The study has several parts and one of them, the analysis of what is offered to the students in the calculus textbook, will be reported in this paper. In another paper the actual use of the textbook by students will be treated and a third paper will investigate some authors' ideas about the calculus textbook. More exploration of what the textbook has to offer to the students and how it is done could contribute to higher awareness in teachers' use of textbooks and to first year's students developing more efficient ways of studying mathematics. Tall (1991, p. 17) writes:

> During the difficult transition from pre-formal mathematics to a more formal understanding of mathematical processes there is a genuine need to help students gain insight into what is going on.

According to Selden and Selden (2001) mathematics education research can not be expected to reveal one "best practice" for how to teach a topic but it "can help develop ways of teaching specific mathematical topics that arise from an understanding of both mathematics and pedagogy" (p. 247). Artigue (2001) also emphasises that it is necessary to improve the links between existing practices, and research about learning mathematics at tertiary level. In particular, future engineering courses could be a possible field of application for the research findings of this study.

Background to the research questions

It is normally expected that students at tertiary level work more individually than students in upper secondary school. Robert and Schwarzenberger (1991, p.128) write: "The students can not learn all new concepts in class time alone. Significant individual activity outside the mathematics class is now an absolute necessity". Much of that work relies on the textbook, which can become an important factor in the process of learning mathematics. In this study we consider the book as a learning tool when students take the calculus course. We search for a holistic picture of what the textbook offers to students. The main focus is on introduction and treatment of the mathematics concept in the calculus textbook used by first year engineering students. The way the concept is introduced to the students is important in the process of acquisition of the concept. Presentation of the concept should encourage interest, create motivation and start the process of learning the concept. The way in which a concept is introduced and treated can also create essential problems for the students. It is of course not possible to investigate the whole textbook in detail. Thus some kind of limitation must be done. We decided to study the chapter that introduces the derivative, because this concept is a basic concept in calculus and understanding of it is fundamental for applications and the future study of other engineering courses. It is one of the first concepts at university level mathematics that is more demanding than earlier concepts, as it is based on the concepts of function and limit, both documented to be demanding (Cornu, 1991; Juter, 2006; Juter & Grevholm, 2007). The concept of derivative is not a new one for the students. According to their pre calculus background, the students should be familiar with it. The concept has been presented in upper secondary school (Oldervoll, Orskaug & Vaaje, 2000)[1]. There it is defined both as a rate of change and as a slope of a curve at a certain point. Analysing the presentation of the concept of derivative by textbooks in the secondary school, we noticed some tendencies to a rather practical approach to the concept. After presentation of the definition, the application of the concept follows very soon. The concept is mainly used to determine some properties of functions, such as whether the function has a maximum or minimum. This fact emphasises the necessity of considering added features in the way the concept is presented and treated in the textbook used during the first tertiary mathematics course. The issue of conceptual and procedural knowledge is also important when considering students' learning of mathematics. Many of the engineering students seem to think that they have to learn only concrete and applied mathematics and not abstract and pure mathematics (Kummerer, 2001).

Thus, in the part of the study reflected in this paper we pose the following research questions:

1. What characterises the introduction of the derivative and the further treatment of the concept in the calculus textbook for first year engineering students?

2. What kind of knowledge does the textbook emphasise?

We wanted to explore the holistic impression of what learning opportunities the book offers to the students. In trying to consider the introduction of the derivative concept in the textbook we studied the context used and the way in which the concept is introduced. Considering the treatment of the concept we focus on emphasis of conceptual and procedural knowledge in the examples and exercises proposed to the students.

Below we will first present some research results relevant for our study and the theoretical framework we have used. The following constructs are part of our framework: mathematics and textbooks, the concept of derivative, the mathematical definition, and conceptual and procedural knowledge. Then we go into the methods used and the methodological considerations. This is followed by analysis and main results and we end by a discussion and some conclusions.

Mathematical textbooks and the concept of derivative

Many research studies about textbooks in mathematics have been done. However most of the research is about textbooks used on lower levels. Pepin and Haggarty (2001) analyse the ways the textbooks are used in classroom contexts and how this influences the culture of the mathematics classroom. Johansson (2005, 2006) considers the textbook as the potentially implemented curriculum. Her focus is on how teachers use the textbook and she concluded that the teachers depended on the textbook. Juter (2006), in her study about students' problems with limits, claims that textbooks used at upper secondary schools do not provide much theory or many tasks in that area, and thus most students do not have a well developed concept image about limits. Because of this the transition from high-school textbooks to university level textbooks can be difficult. A few studies have also been done about textbooks at tertiary level. Raman (2002) discusses difficulties students could have with informal and formal aspects of mathematics when they use pre-calculus and calculus textbooks. Lithner (2004) analyses exercises in different calculus textbooks with a main focus on mathematical reasoning. He also studies students' reasoning when they are working with textbook exercises (Lithner, 2003). Contrary to Lithner, who only investigates the textbook exercises, we intend to explore the entirety of what the book offers in one specific topic. Also, we did not find any studies that answer the questions about what engineering students learning mathematics are offered from a more holistic perspective on the book.

Students' problems with learning of the notion of derivative are explored in well-known and wide ranging previous research (e.g Orton, 1983; Tall, 1992a, 1992b). Orton (1983) showed that students had problems with questions that required explanation of the meaning of the derivative. Some new studies about derivative have also been done. Viholainen (2008) examined informal and formal understanding of the concepts of derivative and differentiability. The study shows students' problems with connecting formal and informal reasoning and in particular that students avoid using the definition of the derivative in problem solving situations. Hähkiöniemi (2006) has developed a model of a hypothetical learning path for the concept of derivative. According to him the learning in the conceptual-embodied world means perceiving the rate of change, local straightness and increase, steepness and horisontalness of a function. Learning in the proceptual-symbolic world (Tall, 2005) could be experienced through calculating average rate of change over different intervals. According to Hähkiöniemi (2006), this creates a natural need for the limiting process of the difference quotient and thus for the formal definition of derivative. The suggested learning path illustrates, as we see it, the need for variation in the learning process and the need to highlight different properties of the derivative.

Theoretical framework

The analysis of the introduction and treatment of the derivative concept presented in the textbook for engineering students is based on some selected theories about learning mathematics. The notions of concept image and concept definition, meaningful learning and conceptual and procedural knowledge are essential in this analysis. Below we present the different theoretical constructs that are of importance in the study.

Mathematics and textbooks

The role of the mathematics textbook seems to be varying according to the different levels in mathematics education. The textbook used at primary and secondary school usually covers the topics defined in the curriculum that students should work with during a particular school year. The textbooks used by students at university level usually cover more topics than those encountered in a single course unit. At tertiary level the curriculum is often given in a short text and the course content is defined by the list of literature. For example, the University college department where the study was conducted emphasises that the mathematics course should ensure a theoretical foundation that can be applied to engineering subject matter and that ensures that students are able to work with professional literature based on mathematics.

The textbook, recommended by the teacher, gives some important messages about what topics are expected to be learnt during the particular course and about the nature of mathematical knowledge. Formal mathematics is represented by definitions, theorems and proofs. Informal mathematics can also use definitions but they are often of a more descriptive character and refer to the intuitive understanding of the concept. The issue of interplay between formal, informal and intuitive aspects of mathematics has been and is still discussed in mathematics education research (Fischbein, 1994, 1999; Raman, 2002; Pettersson, 2008). According to Dreyfus (1991, p. 27) mathematics is often presented to the students as

> the finished and polished product, even though historical mathematics was created through error, intuitive formulations, etc. This way of presenting may work well for students who major in mathematics, but it can be difficult for students majoring in science or engineering and taking mathematics as a required service subject.

The concept of derivative

The concept of the derivative is one of the fundamental concepts in calculus. The concept is particularly important for engineering students because of its application in other subjects. At the same time the concept is complicated; it relies on the limit concept which creates many problems for the students (Cornu, 1991; Juter, 2006). The concept of differentiation is graphical in its origin and was arithmetised in the 19th century through the work of Cauchy, Riemann and Weierstrass.

In the calculus textbooks the concept of derivative is usually first defined at a fixed point as follows

> [...] we considered the derivative of a function *f* at a fixed number *a*:
>
> 1. $f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$
>
> Here we change our point of view and let the number *a* vary. If we replace *a* in equation 1 by a variable *x*, we obtain
>
> 2. $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$
>
> Given any number *x* for which this limit exists, we assign to *x* the number $f'(x)$. So we can regard f' as a new function, called the derivative of *f* and defined by equation 2. We know that the value of f' at x, $f'(x)$, can be interpreted geometrically as the slope of the tangent line to the graph of *f* at the point $(x, f(x))$. (Stewart, 2003, p.165)

Thus the definition consists of two different definitions. First the definition of the derivative in a point and then the definition of the new function f', the derivative function of *f*. These two aspects, local and global, of the definition have to be distinguished when the concept is introduced.

The issue of mathematical definition

A mathematical definition is designed to describe a mathematical idea. The definition can be of formal or informal character. Previous research shows that formal definitions can create serious problems in the concept formation of students (Vinner, 1991; Cornu, 1991; Juter, 2007). To study this issue we find the theory of concept image and concept definition useful. These notions were introduced by Tall and Vinner (1981). They distinguish between the formal definition, often presented in the textbook, and the complete set of ideas that a learner has about a particular concept. This *concept image* is built up from previous experiences of all kinds and can be changed as the individual meets new situations. This draws attention to the issue of previous knowledge of the students that has to be considered, when a new concept is presented. New concepts in many textbooks are presented by a definition (Vinner, 1991, p. 66). To be able to work with the concept students need to achieve a rich concept image. To present the concept by a formal definition is useful only if the definition is meant to be used actively by the students (Fischbein, 1994). Otherwise, the definition will be stored in the memory as an isolated piece of information, not linked to any other conceptual structure. Dreyfus (1992, p. 25) emphasises that it is not sufficient to define and exemplify an abstract concept. Students have to use the definition to construct the properties of the concept through deductions. The definition has to be given meaning in order to be useful. Vollrath (1994) mentions some abilities that help students to develop meaning of the definition: to give examples and counterexamples, to test examples, to know properties, to know relationships between concepts, and to apply knowledge about the concept.

Conceptual and procedural knowledge

According to Hiebert and Lefevre (1986) it is difficult to give a precise definition of conceptual and procedural knowledge: "Not all knowledge can be usefully described as either conceptual or procedural. Some knowledge seems to be a little of both, and some knowledge seems to be neither" (p. 3). Conceptual knowledge is described as knowledge that is rich in relationships. It grows through the creation of relationships between existing knowledge and new information or between two pieces of information that the learner already knows. Ausubel (2000) used the term *meaningful learning*, defined as a process through which new knowledge is assimilated by connecting it to some existing relevant aspects of the individual pre-existing knowledge structure. Other researchers, for example Novak and Gowin (1984), have elaborated on the concept of meaningful learning and emphasise that the students themselves decide if the learning will be meaningful, that is richly connected to the already existing knowledge structures. Hiebert and Lefevre (1986, pp. 7–8) defined procedural knowledge as follows:

> One kind of procedural knowledge is a familiarity with the individual symbols of the system and with the syntactic conventions for acceptable configurations of symbols. The second kind of procedural knowledge consists of rules or procedures for solving mathematical problems. Many of the procedures that students possess probably are chains of prescriptions for manipulating symbols.

Procedures may or may not be learnt with meaning. Procedures that are learnt with meaning are procedures that are linked to the conceptual knowledge. The relationship between concepts and procedures is an important issue in learning of mathematics (Hiebert & Lefevre, 1986; Silver, 1986). Students with only procedural knowledge can receive correct answers when they are working with tasks, but they do not understand what they do and why they are acting in a specific way. According to Fischbein (1994) solving procedures that are not supported by formal, explicit justification are forgotten sooner or later. And Hiebert (2003, p.17) claims that students who practice procedures before they understand them have more difficulties to make sense of these procedures later. On the other hand, students with good intuitive sense for mathematical concepts can have problems with using procedures. It is not enough to understand a system of concepts to become able to use them in solving problems. According to Star (2005) each type of knowledge, both conceptual and procedural, can be either deep or superficial. He considers flexibility, comprehension and critical judgment of use of particular procedures as indicators of deep procedural knowledge (p. 408). Some procedural operations can also contribute to establish more confidence with treating the concept and more conceptual understanding of the concept. Tall and Ali (1996) use the term ”conceptual preparation” to describe some operations or simplifications used in order to make the algorithm easier to apply. Their study shows that the more successful students were more likely to use some form of conceptual preparation.

From our interviews with and observations of students and teachers in the engineering programme (Randahl, 2010) we experience that the teachers are eager to teach for conceptual knowledge but the students are more interested in a quick fix, through learning algorithms and procedures. This makes it important for us to use the theory on procedural and conceptual knowledge as our framework here.

Methods and methodological considerations

This study is an exploratory case study. The textbook studied is ”Calculus – a complete course” written by Robert A. Adams (1991). It has been used in 2006 and 2007 in a basic calculus course for engineering students at the university college. The book is used at several universities and universities colleges in Norway (and in Scandinavia) as a main textbook in basic and more advanced calculus courses. According to the author "The text is designed for general calculus courses, especially those for science and engineering students" (preface, p. xv). We analyse the 6th edition (2006) of the book. The book consists of seventeen chapters and five appendixes. In our study, parts of chapter 2 "Differentiation" have been analysed. The chapter "Differentiation" consists of eleven sections. We analyse the first three of them (pp. 93–113):

2.1 Tangent lines and their slopes.

2.2 The derivative.

2.3 Differentiation rules.

In every section we examine the introduction and treatment of the concept, definitions, examples and exercises which are proposed to the reader.

The analysis started with an exploration of the structure of the sections in the book. We notice that the structure of the presentation of topics in the textbook is almost the same in every section: introduction, definitions and results, examples with some explanations and exercises in the end of the section. The part with exercises is strictly separated from the rest of the text. The three kinds of building blocks, introduction, definitions and results, examples and exercises were further analysed in detail.

Methods for analysis of the introduction of the concept

In the introduction of the derivative in the textbook, we investigate how the concept is presented, what context and kind of definition is used and what previous knowledge is required. We also consider what position the definition has in the treatment of the concept. We make distinction between informal and formal definition. When we use the term formal definition we mean the concept definition accepted by the mathematical community (Tall & Vinner, 1981). By informal definition we mean the verbal explanation of the concept without using mathematical symbols. We consider also how global and local aspects of the concept are treated in the text.

Methods for analysis of examples

With examples we mean mathematical problems presented together with a solution in the text. When presenting a new concept, many textbooks give some examples to illustrate properties of the concept. The formation of a concept requires examples that have something in common in order to notice the characteristics of the concept (Skemp, 1987; Zazkis & Leikin, 2007). The examples can help the student to develop better concept images and add to the students' experiences because in the examples certain aspects of the concept are highlighted. We explore how the local and global perspectives on the derivative concept are treated in the examples.

Some functions are differentiable and some are not and this might not be clear to students. An analysis of whether a particular function is differentiable or not, and if not, why not, can contribute to new experiences of the concept.

The students need both procedural and conceptual knowledge. The textbook should offer examples to illustrate both kinds of knowledge. One way to assist the students to create connections between conceptual and procedural knowledge is to focus more on justification (Fischbein, 1994). It is also important that the students are able to use knowledge in different contexts and situations. Thus, in the analysis of the examples exposed to the students we study the justification aspect and new context aspect, and we use the following categories:

1. Worked examples (only explicit solutions are given and can be used directly to find correct answers when working with exercises; no focus on justification).

2. Examples which intend to increase understanding of the concept (by using different contexts or where justification is required).

To indicate how the categories were used we offer some illustrations of the analysis. A worked example could be like the following (Adams, 2006, p. 95):

> Find the equation of the tangent line to the graph of the function $f(x) = x^2$ at the point (1, 1).

Solution:

> Here $f(x) = x^2$, $x_0 = 1$ and $y_0 = f(1) = 1$. The slope of the required tangent is:

$$m = \lim_{h\to 0}\frac{f(1+h)-f(1)}{h} = \lim_{h\to 0}\frac{(1+h)^2-1}{h} = \lim_{h\to 0}\frac{1+2h+h^2-1}{h} = \lim_{h\to 0}\frac{2h+h^2}{h} = \lim_{h\to 0}(2+h) = 2$$

> Accordingly, the equation of the tangent line at (1, 1) is $y = 2(x - 1) + 1$ or $y = 2x - 1$.

In this example, a particular procedure is given and can be directly "copied", when students work with similar problems in the exercises. Here is an example in the second category (Stewart, 2003, p. 170):

> Where is the function $f(x) = |x|$ differentiable?

Solution:

> If $x > 0$, then $|x| = x$ and we can choose h small enough that $x + h > 0$ and hence $|x + h| = x + h$. Therefore, for $x > 0$ we have

$$f'(x) = \lim_{h\to 0} \frac{|x+h| - x}{h} = = \lim_{h\to 0} \frac{(x+h)-x}{h} = \lim_{h\to 0} \frac{h}{h} = \lim_{h\to 0} 1 = 1$$

> And so f is differentiable for any $x > 0$.

Similarly, it is shown that f is differentiable for any $x < 0$. Further the differentiability for $x = 0$ is considered. The left and right limits are computed. Since these limits are different, $f(0)$ does not exist and f is differentiable at all values except zero. This example needs special justification and is important, because many students assume (as soon as the definition of the derivative is introduced) that all functions are differentiable. It is also crucial to consider special points (here $x = 0$), where f is not differentiable.

Methods for analysis of exercises

Exercises mean mathematical problems given for the students to solve by themselves. We examine them with the intention to locate the emphases on conceptual and procedural knowledge and how the possible links between these kinds of knowledge could be created. Starting from the theoretical framework about procedural and conceptual knowledge we decide to use the three following categories:

1. Exercises which mainly require the use of particular procedures.
2. Exercises which require some conceptual preparation before one can use a procedure.
3. Exercises in which justification of the solution is required or new context is used.

Exercises in the first category are often called ”drill exercises”. They help the learner to develop skills in calculation. The following exercise illustrates the first category:

> Calculate the derivative of the function $f(x) = \dfrac{1}{x^2 + 5x}$.

The student is only expected to follow the specific procedure to obtain the answer.

We assume that the second and third categories of exercises promote the conceptual knowledge and help the learner to develop connections between the concept and procedures. For example consider the following exercises: (inspired by similar examples in Stewart, 2003 and Adams, 2006)

1. Find the points on the curve $y = x^4 - 6x + 4$ where the tangent line is horizontal.
2. For what values of x is the function $f(x) = |x^2 - 9|$ differentiable? Find f' and sketch the graphs of f and f'.
3. Sketch the graphs of the function $f(x) = 3x - x^2 - 1$ and its derivative $f'(x)$. What feature of the graph of $f(x)$ can you infer from the graph of $f'(x)$?

The exercises are not difficult to solve. But some preliminary reflections are required to give the correct answers. In exercise 1 one has to take into account that the tangent has to be horizontal. In exercise 2 the notion of absolute value has to be considered before the differentiation can be discussed. We consider exercise 1 and 2 to be of category 2. Sketching the graphs of both f' and f', and analysing them, in exercise 3, gives the opportunity to obtain better understanding of the derivative concept. Exercise 3 is of category 3. In the investigation of the exercises and examples, we also consider if and how the definition of the derivative is used.

Analysis and main results

Introduction of the concept

The aim of the introduction of the concept of derivative is quite clearly stated in the section. The problem of slopes is defined by the author as one of two fundamental problems which are considered in calculus. Its solution is the topic of differential calculus (Adams, 2007, p. 93).

The author makes no visible connection to students' previous knowledge about the derivative and the way in which the concept could have been introduced in the upper-secondary school. There is no practical context/situation in the text, which clearly points out the necessity of extending existing knowledge about the derivative. The introduction of the concept has a highly formal mathematical character. The definition of the derivative concept is built up developmentally; it starts with a mathematical problem of finding a straight line *L*, which is tangent to a particular curve C at a point *P*. The Newton quotient (also called differential quotient) is introduced and the definition of the tangent and the slope of the tangent are stated in terms of the limits. The definition of the derivative is presented as the limit of the Newton quotient (p. 98):

> The derivative of a function *f* is another function *f'* defined by
>
> $$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$
>
> at all points *x* for which the limit exist (i.e., is a finite real number). If $f'(x)$ exists, we say that *f* is differentiable at *x*.

The concept of the derivative is presented by the formal definition. No informal, intuitive alternatives or graphic illustrations are given. As mentioned previously, the definition relies strongly on the concepts of function and limit and they are known to be difficult for the students. The derivative of a function *f* at a fixed value is not explicitly defined. The given definition starts with the global view, the derivative as a function. Differentiability at one value *x* is mentioned after the global view. The local perspective is also treated in an implicit way as a remark, where two different kinds of notation are exposed (p. 99):

> *Remark.* The value of the derivative of *f* at a particular point x_0 can be expressed as a limit in either of two ways:
>
> $$f(x_0) = \lim_{h \to 0} \frac{f(x_0 + h) - f(x_0)}{h} = \lim_{x \to x_0} \frac{f(x) - f(x_0)}{x - x_0}$$

Further the term of differentiation is introduced as follows (p. 99):

> The process of calculating the derivative *f'* of a given function *f* is called differentiation.

Sketching the graph of *f'* is described in the book as a procedure and is called graphical differentiation. Differentiation is thus related both to algorithmic and graphic treatment. Later in the text the students are guided to do algebraic calculations of derivatives from the definition of the derivative. The derivatives of elementary functions are expected to be memorized. The author writes: ”Derivatives of some elementary functions are collected in table 1 later in this section and are recommended to be memorized" (p. 100). We noticed two ways of relating to the definition:

1. Direct use; for example (ex. 11–22, p. 105):

> Calculate the derivative of the given function directly from the definition of the derivative.

Here the students were expected to use directly the procedure exposed in example 2, p. 100–101. And on the other hand:

2. Indirect use; as for example in the following (ex. 28–31, p. 105):

> Using the definition of the derivative, find equations for the tangent lines to the following curves at the indicated points.

The main aim is to find the equation for the tangent.

Results about examples

There are seven examples in section 2.1, five examples in 2.2 and ten examples in section 2.3. The majority of the examples are worked examples. They mainly present a procedure to solve a problem. It can be illustrated by the following (ex. 7, p. 111):

> Differentiate the functions:
>
> a) $f(x)=\dfrac{1}{x^2+1}$ and b) $f(t)=\dfrac{1}{1+\frac{1}{t}}$.
>
> Solution: Using the Reciprocal rule.
>
> a) $\dfrac{d}{dx}(\dfrac{1}{x^2+2})=\dfrac{-2x}{(x^2+1)^2}$.
>
> b) $f'(t)=\dfrac{-1}{(t+\frac{1}{t})^2}(1-\dfrac{1}{t^2})=\dfrac{-t^2}{(t^2+1)^2}\dfrac{t^2-1}{t^2}=\dfrac{1-t^2}{(t^2+1)^2}$.

and (ex. 5, p. 109):

> Let $y = uv$ be the product of the functions u and v. Find $y'(2)$ if $u(2) = 2$, $u'(2) = -5$, $v(2) = 1$ and $v'(2) = 3$.
>
> Solution: From the Product rule we have
>
> $y' = (uv)' = u'v + uv'$.

Therefore

y'(2) = *u*'(2)*v*(2) + *u*(2)*v*'(2) = (-5)(1) + (2)(3) = -5 + 6 = 1.

Only one worked example points out two ways to solve the problem (ex. 3, p. 109):

Find the derivative of $(x^2 + 1)(x^3 + 4)$ using or without using the Product rule.

But we also identify other types of examples: those which emphasise justification or show other possible contexts in which the concept can be used. One illustration is (ex. 6, p. 110):

Use mathematical induction to verify the formula $\frac{d}{dx}x^n = nx^{n-1}$ for all positive integers n.

Solution:

For $n = 1$ the formula says that $\frac{d}{dx}x^1 = 1 = 1x^0$, so the formula is true

in this case. We must show that if the formula is true for $n = k \geq 1$,

then it is also true for $n = k + 1$.

Therefore assume that $\frac{d}{dx}x^k = kx^{k-1}$. Using the Product rule we calculate

$$\frac{d}{dx}x^{k+1} = \frac{d}{dx}(x^k x) = (kx^{k-1})(x) + (x^k)(1) = (k+1)x^k = (k+1)x^{(k+1)-1}.$$

Thus the formula is true for $n = k + 1$ also. The formula is true for all integers $n \geq 1$ by induction.

Another example (ex. 4, p. 102):

Verify that: If $f(x) = |x|$, then $f'(x) = \frac{x}{|x|} = \operatorname{sgn} x$.

Out of twenty-two examples proposed to the students, we find that seventeen of them can be described as worked examples with emphasis on procedures. Only five examples have emphasis on justification. Emphasis on justification could support development of conceptual knowledge. It seems that the main role of the examples is to demonstrate the use of particular procedures. The students are not challenged to give examples of their own. The difference between the derivatives f' of f at a *fixed value a* and f' as a new function with *x as variable*, is not taken up as a problem to be discussed.

Results about exercises

In total, 140 exercises are proposed to the students in the three first sections. We categorise the problems according to their emphasis on procedural or conceptual knowledge.

Examples of exercises with main emphasis on procedures (category 1):

1. Find an equation of the straight line tangent to the given curve at the point indicated. (ex. 1–12, p. 98)

In order to reach to the expected answer it is only required to use the definition of the slope of the curve. No conceptual preparations are required. The procedure to receive the correct answer is demonstrated in detail in ex. 7, page 97.

2. Calculate the derivatives of the given function. (ex. 1–32, p. 113)

Only use of differentiation rules is required to receive the correct answer. Thus those exercises demand only procedural knowledge from the students.

Examples of exercises which require some conceptual preparation (category 2):

Find the coordinates of points on the curve $y = \frac{x+1}{x+2}$ where the tangent line is parallel to the line $y = 4x$. (ex. 46, p. 113)

Here the students are expected to make some interpretations of the task, like that the tangent line must have slope equal to 4. The exercises are quite easy to answer but one has to take into account some additional conditions and analyse the situation before using the procedures.

Examples of exercises that require some justification (category 3):

1. Show that $f(x) = |x^3|$ is differentiable at every real number x, and find its derivative. (ex. 52, p. 113)
2. Show that the curve $y = x^2$ intersects the curve $y = \frac{1}{\sqrt{x}}$ at right angles. (ex. 48, p. 113)
3. Show that the derivative of an odd differentiable function is even and that the derivative of an even differentiable function is odd. (ex. 49, p. 106)

These exercises are more demanding. Being able to apply the concept of differentiability is required and the issue of the absolute value has to be considered. Even if the carrying out of the solution of example 49 is not difficult, the derivative is used in a new context.

We found that there are 76 exercises (54 %) in the first category. The exercises are either "drill" exercises or exercises which emphasise the application of different techniques.

There are 34 exercises in the second category and only 21 problems of type: "show", "verify", "prove" which require some justification. Some of them are marked with the symbol *, which indicates that they are on a more difficult level.

We did not find exercises which require explanation of the meaning of the derivative concept, like for example "what does it mean that the derivative to the function *f* in a particular point has the value 5". There is rarely a focus on situations where the functions fail to be differentiable.

The set of exercises is graded in a particular way. The exercises which require mostly knowledge of easy procedures for obtaining correct answers are placed in the beginning of the set. Exercises with emphasis on conceptual knowledge are placed later. This fact can contribute to a situation (for example little available time) in which a majority of the students never work with more challenging tasks. The observations of how students work with the textbook (from the other part of this study) confirm the statement that many students never work with the tasks in the end of the exercises section (Randahl, 2010).

We find that the textbook emphasises learning of algorithms and procedures, which seems to be what the engineering students prefer. For a more long-lasting and substantial learning outcome there is a need for more emphasis on conceptual learning in the textbook and more varied examples and exercises, which illustrate the properties of the derivative in a richer way.

Discussion and conclusion

For the teacher, the textbook offers a source of aspects to teach, of examples to go through and of exercises to ask students to work with. From our interviews and observations in class we know that this also happens (Randahl, 2010). Calculus is quite different from the mathematics the students are used to from before. To give the students an overview of the main ideas of calculus, having more focus on the connections between ideas could be useful. The text could explain better the necessity of an introduction of the derivative concept, which would contribute to improve the motivation to learn the concept with understanding, in order to later use it in different fields of application. When the students do understand and master the main concepts of calculus like limit, continuity and derivative, they ”will have established the foundation for a great deal of very useful mathematics” (Martin, 1969, book 7, p. 1). The book by Martin illustrates a quite different approach than the one we found in Adams' textbook.

The issue about presentation of concepts using different kinds of mathematical knowledge should be a main concern in future research about textbooks. As mentioned before the concept of derivative is not new to the students. But the students entering the basic calculus course in engineering education have rather poor concept images about the derivative (Randahl, 2010). It is mostly created by a procedural approach to the concept in the upper-secondary school. The research questions of this study show that our aim was to investigate the presentation and treatment of the derivative concept in the textbook. We find the presentation of the derivative concept offered to the students very formal. The introduction is clearly mathematical and depends on students' knowledge of the limit concept. There is no practical context or situation which explains the necessity of extending the existing student knowledge. Strict, pure mathematical contexts can contribute to the fact that students see the textbook as hard to use. The formal approach to calculus is discussed in different research papers on mathematics education (Tall, 1986, 1991; Cornu, 1991). Cornu (1991, p. 165) pointed out the problem of context in which the learning is taking place. The students have to see the concept as a useful tool and not only the presentation of a new concept by definition, a sequence of examples and exercises. In the textbook by Adams some examples are given to show applications of the derivative to represent and interpret changes and rates of changes: velocity and acceleration, dosage of the medicine and economics (for example marginal cost of production). But they are considered later in the section 2.11 and in chapter 4: ”Some applications of derivatives”. To point out earlier in the text the application aspect of the concept could make it more interesting for future engineers. Presentation of the concept through discussing (not necessarily very complicated) problems from different fields like physics, economics, and biology could create more motivation and interest for the concept. The emphasis on previous knowledge seems to be an important issue for the author. In the preface Adams stated ”[...] success in mastering calculus depends on having a very solid basis in pre-calculus mathematics (algebra, geometry, and trigonometry) to build upon” (Adams, 2006, p. xiii). It means that the author of the book has some expectations of the students' knowledge.

But the author makes no reference to the ways in which the derivative might have been treated in the upper-secondary school. To help students to make connections between their previous knowledge and the new mathematical ideas is one of several challenges for the book. The role of the definition in the book is explicitly pointed out in the following way:

> As is often the case in mathematics, the most important step in the solution of such a fundamental problem [to find a tangent line to a curve at given point] is making a suitable definition.
>
> (Adams, 2006, p. 93).

But the introduction of the concept by using a formal definition (and some examples) is not enough to support students' learning. We claim, with Fischbein (1994) and Vollrath (1994), that the definition should be used more actively in the process of concept formation. We find that both examples and exercises have a strong focus on using procedures. In this way the textbook emphasizes the procedural knowledge more than the conceptual. To achieve meaningful learning and build a rich concept image the students can be helped by for example working with different kinds of exercises, which highlight different properties of the concept.

The specific structure of the part with exercises (with more demanding exercises at the end) does not make it easy to work with "justification" tasks. It requires that the students use the textbook in an efficient way and this can be difficult for first year students or that the teacher explicitly guides them. Ability to use rules correctly is important for engineering students. But equally important is to have learnt to use them in different contexts and to know exactly why a particular procedure is needed. It is also important that the students develop some procedural flexibility when they work with tasks (Star, 2005). By proposing tasks which require more than one way of solving the problem, the students could be challenged to make the choice and be more creative. Solving mathematical problems by using an appropriate approach and strategy and evaluating the proposed solution not only require but also contribute to develop a richer concept image. Thus, turning back to our research questions to summarise:

> What characterises the introduction of the derivative and the further treatment of the concept in the calculus textbook for first year engineering students?
>
> What kind of knowledge does the textbook emphasise?

We find that the introduction of the derivative is formal and purely mathematical with few signs of motivation or explanation of the background of the concept. Applications are not given in the introduction, and we find no intuitive explanations that could help students' reasoning. The author does not help students to overcome for example the problem to see the difference between the derivative as a function and the value of that function for a given value of the variable, which is important in order for students to create a rich concept image. The further treatment has an emphasis on procedures and memorisation and the worked examples are straightforward and easy. Few of the examples and exercises support conceptual development and knowledge and students are not challenged to justify, prove or reason more deeply using the concept. The concept images of the students are not given much opportunity to be expanded. We conclude that the textbook has much potential to be improved to meet the needs of students' meaningful learning of mathematics.

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Notes

1 Reference is given to the textbook which is most frequently used in the upper secondary schools in Norway.

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Article 2

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ORIGINAL ARTICLE

First-year engineering students' use of their mathematics textbook - opportunities and constraints

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Abstract The role of the mathematics textbook at tertiary level has received limited exposure in previous research although it is likely that students work individually and that some of this work depends on the use of the textbook. The aim of this study was to investigate the process of approaching the textbook from epistemological, cognitive, and didactical perspectives. The focus was on identifying and discussing the opportunities and constraints in the process. The study was an explorative case study and the participants were first-year engineering students taking a basic calculus course. The data were collected through questionnaires, observations, and interviews. Results showed that the textbook was used to a very low degree and mainly perceived as a source of tasks. Different opportunities and constraints are pointed out and some didactical implications are suggested. The results and discussion indicate that a need for greater awareness about the use of mathematical textbooks in meaningful ways at tertiary level.

Keywords Mathematics textbook · Calculus · Engineering students · Constraints

Introduction

Over the past two decades there has a rapid increase in the amount of research into mathematics education at tertiary level (Holton 2001; Niss 1998). However, issues associated with the use of the mathematics textbook at this level have received limited exposure in previous research. It is expected that students work individually

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more frequently at the tertiary level than in upper secondary school. As Wood (2001) stated:

> Students have trouble coping with large amounts of new material in a short time. Academic staff seem unapproachable and there may be little support for students with difficulties. Students are expected to do much of the work by themselves. (p. 93)

Some of the students' assumed individual work may rely on the use of the textbook and on how the book is used. Because of this it is reasonable to ask: How do first-year students perceive and approach the mathematics textbook? What opportunities are offered and what difficulties can arise?

This study aims to explore first-year engineering students' use of the calculus textbook by identifying the support and difficulties they experience when they start to study the concept of the derivative. According to Artigue (2001), one of the goals of research on mathematics learning and teaching at tertiary level is "to improve the understanding of students' difficulties and the dysfunction of the educational system" (p. 207). The results of this study might expand the understanding of what really happens by giving some insight into students' activities. It might not only make both teachers and students more conscious of the possible problems and help them face these problems more effectively, but also indicate how to take advantage of the existing opportunities. Finally the results may be interesting to authors of tertiary-level textbooks. More knowledge about students' perception of the textbook may inform decisions about introduction and treatment of concepts. The following research questions were posed:

1. What characterises first-year engineering students' approaches to mathematics textbooks?
2. What possible opportunities and constraints influence the ways textbook are approached by students?

Three perspectives on the process of approaching the textbook

The process of approaching the textbook is complex. The student with her previous knowledge, experience, and ideas about mathematics and learning mathematics makes the first evaluation and decisions about further use of the textbook. The process takes place in the context of a certain didactical environment with a given curriculum, and is influenced by the extent to which the textbook content is explicit, and the teacher's vision of how the textbook should be used. Considering the process of learning, Artigue (1994, p. 32) describes the following types of constraints:

1. The *epistemological nature* linked to the mathematical knowledge at stake, the characteristics of its development, and its current way of functioning
2. The *cognitive nature* linked to the population targeted by teaching
3. The *didactical nature* linked to the institutional functioning of the teaching

These three perspectives were found valuable when discussing the opportunities and limitations arising when the students approach the textbook. Within the epistemological perspective the focus will be on how presentation of mathematical knowledge in the textbook and students' ideas about learning may have implications for the ways in which the book is perceived. Within the cognitive perspective the main focus will be on identifying students' cognitive barriers with emphasis on their previous knowledge. Within the didactical perspective the main focus will be on how the textbook is embedded in the calculus course and how the students are expected to use the textbook.

The epistemological perspective

The nature of mathematical knowledge at tertiary level differs from that at secondary level. The knowledge is based more on the formal definition and formal proofs of theorems related to the main concepts (Tall 1991). Raman (2002) conducted an epistemological analysis of how pre-calculus, calculus, and analysis texts treated the notion of continuity. She concluded that the texts send conflicting messages about status and purpose of mathematical definitions. Results obtained from analysis of the textbook used by the students in this study (Randahl and Grevholm 2010) showed that the book promotes formal mathematics.

The concept of a derivative in real-variable calculus is clearly mathematical and student understanding of it at tertiary level is linked to their knowledge of the limit concept. The absence of practical contexts or situations which could point out the necessity of extending the existing student knowledge and the strict, pure mathematical contexts can contribute to the fact that students see the textbook as hard to use (ibid; p. 23). The epistemological perspective refers also to students' ideas about mathematics and learning mathematics. Students generally consider mathematics as "a collection of procedures to be used in order to solve some typical questions given in some crucial exams" (Vinner 2007, p. 4). Many engineering students are not primarily interested in mathematics but admit that mathematics is important in engineering contexts. At the same time the students think that they have to learn only concrete and applied mathematics and not abstract and pure mathematics (Kummerer 2001).

The cognitive perspective

This study takes a constructivist approach to students' learning. The focus is on the learner as an individual who constructs her own knowledge often in an interaction with the others and within an institutional environment. By taking the constructivist perspective on learning it is assumed that the textbook's presentation and treatment of the concept in the textbook are not commonly perceived, with each student makes making her own interpretation. Students using the same textbook can make totally different interpretations of the text. Students' previous knowledge and earlier experience are important when they approach the textbook with the aim to learn mathematics. The calculus course for first-year students is considered to be quite difficult. According to Eisenberg (1991) there is quite a big difference between mathematics in high school and calculus: New concepts need to be learned quickly, and there are many generalisations, abstractions, and formalisations. Tall (1991) emphasised what he called the "cognitive expectations":

> The move from elementary to advanced mathematical thinking involves a significant transition: that from describing to defining, from convincing to proving in a logical manner based on those definitions. This transition requires a cognitive reconstruction which is seen during the university students' initial struggle with formal abstractions as they tackle the first years of university. (p. 20)

Previous research has pointed towards a gap that exists between *concept image* and *concept definition*, and it has been argued that this can be a problem when students learn mathematics (Tall and Vinner 1981; Juter 2006). The *concept image* is all the mental pictures, properties, associations, and processes related to a given concept. It can change with new situations and experiences. Only a part of the concept image may be evoked at a particular time and that part is called an *evoked concept image*. The *concept definition* is a "form of words used to specify the concept" (Tall and Vinner 1981, p. 152).

Students can form their concept image through different examples of the concept. It is expected that the students entering the calculus course have concept images based on earlier experiences and that these interact with the more formal definitions that are presented. The concept of the derivative should not be new for the students. According to the goals stated in the curriculum for the course in upper secondary school, the pupils should have knowledge about average and instantaneous rates, be able to find approximate values for instantaneous rate by calculation, and be able to recognise and interpret examples of instantaneous rate in practical problems.

The didactical perspective

Generally, the curriculum describes the mathematical knowledge that is to be learnt by students. It usually specifies learning goals, content, methods, and assessment procedures (Tietze 1994). The overall goal for engineering education in Norway is centrally stated in the *Rammeplan* [1](2005) in the following way: "To educate engineers who combine theoretical and technical knowledge with practical proficiency and who take the responsibility for interaction between technology, environment, individuals and society" (p. 3). The university colleges at which future engineers are educated formulate more subject-specific *core curricula*, with goals, content, assessment forms, and a list of literature. For example the university college where the study was conducted defined the following learning goals for mathematics courses

1. to ensure a sound theoretical foundation that can be aptly applied to engineering subject matter
2. to contribute to giving the students a solid basis/foundation for further specialisation and post-qualifying education
3. to ensure the same quality standards as in international education programmes

[1] *Rammeplan*=studyplan

4. to ensure that students are able to work with professional literature based on mathematics, and that students develop a language conducive to communication in the technical-scientific environment

Both the study plan and the specific core curriculum express the importance of theoretical foundations and strongly emphasise the applicability of mathematics. It is expected that the future engineers will acquire sufficient mathematical knowledge and skills to enable them to identify, analyse, and resolve engineering problems.

The multiple-perspectives methodology

The setting for the study

The engineering students involved in the study reported here took the *Mathematics 1* course during the first year and *Mathematics 2* later. *Mathematics 1* comprised calculus and linear algebra, but in this study only the calculus part is considered. The mathematics course is compulsory and the normal minimum prerequisite is the completion of an advanced mathematical course in upper secondary school. Some of the students receive the required "study competencies" by taking an intensive mathematical introduction course at the university college. Because of these different preparatory experiences the level of students' previous knowledge can vary greatly.

The calculus section covers topics from differential and integral calculus such as functions, the concept of derivative, rules for differentiation, applications of derivatives, integration, and differential equations. The textbook used by the students in the study reported here was recommended by the teacher for the course. There were many available calculus textbooks, most of them in English, that covered the basic concepts named in the curriculum. The year when the study was carried out the textbook *Calculus* by Adams (2003, 2006) was used. According to the author, this text was designed "for general calculus courses, especially those for science and engineering students" (p. xv).

The investigation took place at one of the university colleges in northern Norway. All 90 participants were first-year engineering students who were taking the compulsory *Mathematics 1* course that comprises calculus and algebra. The textbook, *Calculus – a complete course*, written by Robert A. Adams, was used in the calculus part of the course.

Forms of data: multiple perspectives

The data for the study were collected by a questionnaire given to the students, an interview with the teacher, observations of lectures and task-solving sessions, interviews of three students, and some informal conversations with students (mainly during task-solving sessions).

The questionnaire. At the beginning of the calculus course the students were administered a questionnaire consisting of clearly mathematical questions and questions about students' ideas about learning mathematics and learning sources. The main aim was to obtain insight into students' previous knowledge and to get some idea of students' assumed choice of learning sources and their ideas about learning mathematics. For example, in one section of the questionnaire the students were asked to make sense of the definition of the derivative and use it further. Students were required not to state the formal definition of the derivative, but only to explain how they understand it and to use it to prove an easy rule.

The following excerpts from the questionnaire (translated from Norwegian) offer the reader an idea of the kinds of information sought through the questionnaire:

Question 1 The more "formal" definition of the derivative was introduced in the upper secondary school.

We repeat it here:

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

Explain how you understand this expression and then use it to show that $(x^2)' = 2x$

Question 2 a) The following function $f(x) = x^2 + 3$ is given. The derivative f′(x) to f(x) will be 2x.

Explain what the derivative $f'(x) = 2x$ tells us about the function f.

b) We can find the value of the derivative for a particular x; for example for x=4 we get $f'(4) = 2 \times 4 = 8$. What does the number 8 tell us?

Question 3

What do you assume to be most important / helpful when you learn calculus:

- ☐ *Textbook*
- ☐ *Lecture notes*
- ☐ *Help/discussion with the teacher*
- ☐ *Cooperation/discussion with other students*
- ☐ *Other*

Which? ___________________________________

Question 4

What do you consider to be most important when you learn mathematics?

- ☐ *Understanding*
- ☐ *Own interest*
- ☐ *To see how mathematics can be used*
- ☐ *To get right answer*
- ☐ *Formulas and methods*
- ☐ *Modelling*
- ☐ *Other reasons*

Which? ___________________________________

Interview with the teacher. The interview with the teacher was conducted in the beginning of the term. The purpose of this interview was to discover reasons for the choice of the particular calculus book, and to gather information on the teacher's experiences in using the textbook.

Interviews with students. The choice of three student interviewees was based mainly on observations of the classes and on some informal talks with the students. During class observations one of the interviewees seemed to refer to the textbook frequently, but the other two interviewees rarely referred to the text. All three interviewees performed quite well in tests and during the task-solving sessions. One of the students was female, the two others were male and one of them was from an Asia nation.[2] The student with foreign background was speaking Norwegian at a level which made it possible to follow the lectures and participate in task-solving sessions.

During the interviews with the students, the interviewees responded to a set of questions about mathematics and about the textbook that was used. They were also asked questions about one of the tasks in their textbook. The interviews were audio-recorded and transcribed. The early questions during the interview inquired about interviewees' attitudes to mathematics, for example: Why, in your opinion, do we have mathematics in engineering education? Are you interested in mathematics? Do you like mathematics? Have you had problems in mathematics before?

The questions about using the textbook were as following: Do you use/not use the book during the course? What are the reasons for this? What expectations of the book do you have? What are you looking for? How do you perceive the book: difficult, easy? If difficult, what is difficult? If you do not use the textbook, what do you use: other books, lectures notes?

Observations of lectures and task-solving sessions. Observations of lectures and task-solving sessions took place over a period of six weeks, the aim being to find out more about how the textbook was used by the teacher and the students. For each observation there were about100 students in the lecture hall, and during the group-work sessions most of the students worked in groups of three or four people. However, some students worked alone. During the observations the writer paid particular attention to the extent to which teacher followed textbook approaches during the lectures, and to any references she made to the textbook. Of interest was whether the students were encouraged to use the textbook and, if they were, how. A specific question was: Were the exercises that the students were asked to do taken from the textbook? If they were not, then, where were they from?

During the task-solving sessions the writer was interested in whether the students used their textbook when attempting the tasks. If the answer was "Yes," then how did they use the textbook? Did the students help each other while working in small groups? What kind of questions did they ask each other, and where did they get their answers from? Were sources other than the textbook consulted?

[2] The university college traditionally enrols many students from Asia.

Other methodological considerations

A common concern across all of the data collection techniques was the possibility of loss of objectivity (Lester and Lambdin 1998; Bryman 2004; Golden 2006; Schoenfeld 2007). Could we trust the results? Did the students tell the truth? The choice of methods, of course, depended on the research questions that needed to be answered.

By building into the research design a variety of sources of data it was expected that triangulation of analysis would be facilitated. However, as with all methods, there are advantages and limitations. It was important to consider how data gathered from any one vantage point verified or contradicted information obtained from other vantage points (Wellington 2000; Golden 2006). In this study the questionnaire, observations, interviews, and informal talks were the main methods of data collection. The aim of the questionnaire was to obtain insight into students' previous knowledge and to discover whether the students intended to use the textbook and also what they considered as important when learning mathematics. Analyses of responses to the questionnaire, together with observations, were expected to make it easier to choose students for possible interviews. Interviews might possibly give more information and a better picture of how the book was perceived by the students.

There were several potential limitations associated with the use of the questionnaire. Questionnaire responses revealed only what the responding students wanted us to know. One can assume that they were honest with their answers. But there can be a gap between stated and actual behaviour. With respect to the students' answers to questions on pure mathematics, if no answer was given to a question it would be difficult to know whether the students were unable, or unwilling, to answer the question. Golden (2006) warned that students' responses to questionnaires or interviews may be affected by their desire to present themselves as responsible, especially to researchers who belong to the college staff. In this study the students were encouraged to reflect on any problems they had with perceiving the book as a possible learning source. It has been confirmed by observations that they experienced some difficulties when using the textbook.

The researcher's role as an observer was also considered. One of the issues I faced was whether I should participate in group discussions and help the students with the tasks. In this case participation had to be considered in terms of the possible influence on results and the conclusion. I considered the possibility that it would be easier to be a participant observer, because it might have offered me more contact with the students. But if I had done that, there would have been the possibility that the "natural setting" would have been disturbed by my presence (Leder and Forgasz 2002). Another possible problem was that the students would want quick hints and direct help on how to solve the tasks and the "*how to*" might have become more important than the "why." Being "outside the groups" might help to have more focus on what I would observe.

Another issue was concerned with observations of the lectures. The teacher knew in advance that the research would be about textbooks. How might this information have influenced her behaviour? Would she refer more often than she usually did to the textbook during the lectures? This thought caused me to reflect on the possibility of presenting a more general aim of the study (for example, "Students' difficulties with calculus learning") to those who would be generating the data.

The above reflections and considerations commented on possible limitations of the study. The limitations were identified for two main reasons: first, they made the writer more aware of limitations when the data were being analysed; second, the conclusions reached could be considered in the light of the limitations, which could influence research designs and methodologies for future related studies.

Results

The data comprised responses to the questionnaire (50 students answered), transcripts of interviews with the teacher and 3 students, written notes on observations of 20 lectures and 10 task-solving sessions, and informal conversations with students.

Responses to the questionnaire

Students' responses to the mathematical part of the questionnaire were analysed in terms of their concept images of the derivative concept. Both observations and interviews were transcribed and the outcomes were categorised taking into account the epistemological, cognitive, and didactical perspective. Then responses were analysed in terms of *support* and *difficulty*. In order to find and discuss possible connections, these results were also related to the textbook (Randahl and Grevholm 2010) and to the curriculum.

In response to the question about what learning sources students primarily used when studying, 55 % of the students indicated lectures notes and 35 % indicated the textbook. In response to the question about what is really important when learning mathematics, 78 % chose "understanding" and 14 % answered "correct answer."

Analyses of students' responses to the mathematical tasks on the questionnaire suggested that most students entered the calculus course with poor knowledge of related mathematical content.

Regarding responses on task 1a (making sense of the definition)

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x};$$

- 30 of the 50 students who submitted responses to the questionnaire did not answer this particular question;
- 16 students stated that they could not remember this, or they had never seen anything like it before; and
- 4 students made an attempt to answer, mostly providing explanations of the symbols used in the definition.

Regarding task 1b: using the definition to derive the formulae and *to show* why $(x^2)'=2x$

- none of the 50 respondents answered, or commented on the question

Regarding responses on task 2a, asking for an explanation of what the fact that the derivative $f'(x) = 2x$ *says in relation to the function* $f(x) = x^2 + 3$

- 18 of the 50 questionnaire respondents did not answer this question;
- 15 students gave an incorrect answer that referred to maximum or minimum turnings points;
- 11 gave answers related to the notion of slope;
- 4 gave answers relating to the concept of "rate of change"; and
- 2 commented that it meant that the *f* was differentiable.

Regarding responses on task 2b, which was concerned with the meaning of

$$f'(4) = 2 \times 4 = 8$$

- 18 of the 50 questionnaire respondents did not answer the question;
- 5 students gave answers related to the slope of the tangent at a point;
- 4 students gave an answer related to "rate of change";
- 14 students indicated nothing more than a value of $f'(x)$ had been found; and
- 9 students gave an answer that was unrelated to the concept of a derivative.

Seen from a cognitive perspective, the responses to the questionnaire suggested that the students had poor previous knowledge of differential calculus at the beginning of their course. They were unable to make sense of the definition, and showed no evidence of being able to use it to justify a basic rule. They had difficulties with interpreting meanings of derivative function concepts. When it is recalled that 50 students did not respond to the questionnaire, it is possible that the actual overall situation was worse than that suggested by the above analysis.

Interview with the teacher

According to the teacher, the textbook used in the calculus course had been used for the course for only a few years. Mathematical errors in the text previously used and student complaints about difficulty level were the main reasons given for the change.

According to the teacher two issues were of particular interest during discussions on which textbook should be adopted as the new text for the course: first was clarity of the presentation and treatment; and second was the number of tasks required of students. Before the final decision, two different books were considered but finally *Calculus* by Adams (2006) was chosen. The deciding factor was that the other mathematicians had been satisfied when using Adams textbook with their courses. The following excerpt from the interview with the teacher is pertinent:

> I have looked at and considered many textbooks. You cannot judge them before you have used them, you know. Many had huge amount of tasks, diagrams. I could not recommend them. So finally we [the staff] considered two books and

> after some recommendations from other mathematicians we decided to use this one. [*Calculus* by Adams (2006)]. The book seemed to be well arranged and offered a lot of different tasks to engage students.

When talking about experiences from the year before, the teacher admitted that many students had perceived the new book as difficult, especially its level of formal mathematical language. Because of this, she spent much time explaining the subject matter during the lectures:

> We teach engineering students, not mathematics students, you know. So they have very poor understanding of the mathematical language. This is very difficult for them. So I have to take it on the blackboard.
> The applications are important to them….and I have to use additional examples, not only from the book. You know, the meaning with the textbook is that the student should read it…work with it by herself. But it is not possible. The textbook is huge but it includes many things that are not of our concern.

Observations of lectures and task-solving sessions

The lectures had a clear and careful structure. The teacher used her own notes and wrote everything on the blackboard. She followed the sequence of topics in the textbook, but she did not always use the definitions or examples from the book. It was obvious that she prepared the presentation using several sources (other calculus textbooks, for example). She was a capable and experienced lecturer with very good subject knowledge. She had a friendly attitude to the students and they were willing to ask her questions during or after the lectures. The students made lecture notes. Many students had the textbook on the desk but they looked at it only when the teacher made direct references to the book. For example the teacher said:

> This definition […..] and other examples you can also find in the book. You can look at this later [at home].
> [*Many students opened the book and checked if the reference was correct*]

At the end of every lecture the teacher wrote on the blackboard a list of exercises recommended for the next task-solving session. Approximately 90 % of the tasks were selected from the textbook. The students knew the content plan of the lectures but they were not encouraged to read the text in the book in advance. During the task-solving sessions most of the students were sitting in groups with 3–4 people. Students appeared to be quite motivated during the task-solving sessions. Many students spent most of their time studying examples from the book, trying to apply them to obtain correct answers, but they were uncertain how to start to solve the problems. They were turning over the pages in the textbook, and it seemed that they were not very familiar with the book. Some of students were working in a special way; they started with the first exercise in the exercise section in the book (even if it is not the "homework"). Students were asked for the reasons why they chose to work in this way. They answered that mathematics was difficult for them and they had to start with the easy exercises and go forward to the more difficult ones. But there were many exercises in each section (approximately 60), the time was limited, and the work was not finished. The result was that students were very frustrated at the end of the sessions. Many students were waiting to get the help from the teacher. Here is one observed episode:

> Two students, who were sitting together, were trying to work on the task.
> Student 1: ….what shall we do here?
> Student 2: I am not sure…, look in the book, maybe we can find something similar…
> [*one of the students looks in the book, the other looks at lecture notes. They turn over the pages; no one looked at the pages with theory. It takes approximately 12 min. They do not talk to each other*]
> Student 1: Ok……maybe this one….no, not similar…
> Student 2: Maybe look at the answers…
> [*He looked at the answer section in the textbook*]
> Student 1: … no, nothing,…..only short answer….
> Student 2: we have to ask the teacher…..
> They try to contact the teacher. He is quite busy with helping the other students. Student 1 and student 2 are sitting and waiting. They do not try to work any more. At last the teacher came to them and asked what the problem was. They said that they had tried to work with the exercise but it was too difficult. The teacher explained the problem, drew the situation from the exercise on the paper and gave some hints to solve the problem. The students started to work; they did not talk any more.

The outcomes of the observations were mainly of didactical nature and they gave a picture of how the textbook was used by the teacher during the lecture and by the students during the task-solving sessions.

Interviews with the students

Two of the students were Norwegian; the third one was from Asia. All three agreed that the mathematics was important for future engineers. The three interviewees regarded the tasks as particularly important. The following comments were typical:

> All engineers have to study mathematics. But not so much theory - I mean definitions and theorems. I cannot see how it can help us to understand ….mathematics.
> You know, the tasks are very important for engineers. It is all about the tasks. We spend most of the time working on them.

The Norwegian students perceived the textbook as very difficult to use. Both of them liked mathematics and one of them was doing very well at the tests. During secondary schooling they had become accustomed to reading mathematics textbooks. Because of these experiences they both selected the textbook as one of the main learning sources when they answered the questionnaire. But according to them the book they had to use for this course was too difficult. One of the students had tried to read the text in the beginning of the term but that been a frustrating experience. Here is the relevant excerpt from the interview:

> Student: I read the first chapter [Preliminary] and the two following chapters in the book before I gave it up. Now I use it only when I have some problems. It is much easier to use the lecture notes.
> Interviewer: What kind of problems?
> Student: When I had problems with the tasks.

Another interviewee stated:

> I look through examples….beyond that it is so much talk…the book has so much text, it makes it so difficult. And so compact…I use it only if it is absolutely necessary, for example when the exercises given by the teacher are from the book…

Regarding differences between textbooks at secondary and tertiary levels, one student said:

> It was easier to find what I looked for, it was easier to understand, easier to read, with easier examples.
> And it was in Norwegian, because of these things, [it] was easier.

Excerpt from the interview:

> Interviewer: When you have problems, what do you do: ask anybody to help, use the book or the lecture notes?
> Student: I do this [use the book and the lecture notes] when I sit alone, but at school it is easier to ask for help because one can get an explanation.
> Interviewer: What about explanations in the book?
> Student: They are not so good,….I mean the teacher explains better.

Both of the Norwegian interviewees said that the book was not much used in lectures. In their opinion it was possible to perform well at the tests by using the lecture notes only. They said that by studying the lecture notes they got the necessary understanding of the calculus. The student from Asia had quite a different opinion about the textbook. First of all she was used to reading the book before the lecture.

The following was a comment made by the Asian student in an informal talk during a task -solving session:

> I have to read it in advance. In this way I can think more deeply when I follow the lectures. By doing this I find it easier to follow what the teacher is writing and to know what he means.

She explained that this was the way she had got used to working when studying mathematics in Asia. She was adamant that for her this was the only way she could obtain an understanding of important calculus concepts. She perceived the theory in the textbook as important and wished that she had more time to read through all the definitions and examples.

> I read the theory first, and after this I read examples to understand better the theory. The theory is most important, you know, so I read it first and after this the examples, and tasks… and once more the theory….The book is good, it is well organised. There are a lot of examples, tasks, …and theory. But it is in English. Sometimes…I have difficulties with the language.

Discussion

The aim of this study was to find out what characterises first-year engineering students' approaches to using the calculus textbook. The assumption was that in the process of approaching the textbook some opportunities and limitations were present and that they could be recognised. The process was considered from epistemological, cognitive, and didactical perspectives.

Analysis of the data indicated that the students preferred to use the lecture notes rather than the textbook. During the term the students seemed, increasingly, to ignore the textbook except in relation to the tasks offered by the book.

Why did the students lose interest in the textbook? One of the reasons seemed to be that the formal treatment of the concepts in the textbook was too difficult for the students. Other researchers have also reported that ways in which mathematics is presented in textbooks can present real difficulty for students. Kajander and Lovric (2009), for example, when considering the source of students' misconceptions, suggested that more attention should be paid to the presentations of mathematical concepts in textbooks. According to Dreyfus (1991) mathematics is often presented to the students as

> the finished and polished product, even though historical mathematics was created through error, intuitive formulations, etc. This way of presenting may work well for students who major in mathematics, but it can be difficult for students majoring in science, engineering and taking mathematics as a required service subject. (p. 27)

Lakatos (1976) drew attention to the "deductivist style" of presenting mathematics:

> This style starts with a painstakingly stated list of axioms, lemmas and/or definitions. The axioms and definitions frequently look artificial and mystifyingly complicated. One is never told how these complications arose. (p. 142)

And Alsina (2001) stated:

> Mathematics courses present positive results, solved problems, bona fide models. Students become convinced that mathematics is almost complete, that theorem proving is just a deductive game, that errors, false trials, and zig - zag arguments, which play such a crucial role in human life, have no place in

> the mathematical world. Unfortunately, in some ways many textbooks have inherited the cold-journal style. This style of presentation kidnaps the 'human nature' of mathematical discoveries, the mistake that were made, the difficulties and the need for simplification. (p.5)

The following question arises: Did the first-year students using the textbook have any particular expectations of the textbook?

Sosniak and Perlman (1990) pointed out the relation between cognitive demands and students' prior knowledge so far as textbook usage was concerned:

> The cognitive demands of textbooks cannot be analysed without paying attention simultaneously to the prior knowledge and experience of the student who will use the book and the uses to which the book will be put. (p. 440)

Lakatos (1976) noticed:

> Some textbooks claim that they do not expect the reader to have any previous knowledge, only a certain mathematical maturity. This frequently means that they expect the reader to be endowed by nature with the ability to take a Euclidean argument without any unnatural interest in the problem-background, in the heuristic behind the argument. (p.142)

This study showed that first-year engineering students when starting the calculus course experienced serious difficulty not only in making sense of textbook definitions but also in using them. Their textbook introduced important concepts through formal definitions, and hence students found that it was not much help trying to use the textbook as an aid to understanding. The formal language used by the textbook was clearly perceived by the students as something that made the theory incomprehensible for them. It seems to have been the case that the gap between the students' previous knowledge and the expectations represented in the presentation of mathematical knowledge in the textbook was too large. The above discussion resonates with Zevenbergen's (2001) statement that having "access to the formal language of instruction and text, students' progress is enhanced or impeded depending on their levels of familiarity and competence in the language of instruction" (p. 15).

Taking a didactical perspective raises the question of the role of the textbook in the calculus course given by the particular educational institution. The explicitly defined goals of the core curriculum might be regarded as possible affordances in the process of approaching the textbook. They gave the students opportunity to define their own aims with the course and to relate them to the textbook. But because the goals of the curriculum were not clearly related to the use of the textbook by the teacher, the students found that they could easily ignore the book. Additionally it was necessary to take into consideration how mathematics textbooks were used in secondary mathematics classes. In fact, when they had been at school, most of the students in the present study had not made much use of mathematics texts. Apparently that is not uncommon, for as Sosniak and Perlman (1990) wrote:

> The textbook is seldom used as the source for insight into strategies for the solution of the problem or for explanation or for clarification of the concepts underlying the problems students are asked to solve. Occasionally the students report being expected to read the narrative portion of the mathematics text ('the sides of the pages'), but it is true only in a small number of instances. More importantly, perhaps students rarely are encouraged to study this narrative seriously. Instead, they count on teachers to 'explain it correctly'. (p. 429)

In the interviews it was confirmed that little experience with reading mathematics text in the textbook at secondary level was one of the reasons that the tertiary students found it difficult to use their mathematics textbook.

This study also called attention to the different ways in which the mathematics textbook is embedded as a teaching tool at primary, secondary, and tertiary levels. Whereas school teachers of mathematics, especially at the primary level, tend to depend very much on the textbook (Johansson 2006), at the tertiary level teachers often perceive themselves as experts in mathematics. In the present study, the teacher gave lectures which she had prepared, and the lectures she gave were interpreted by the students as representing their teacher's own knowledge.

However, this perceived expertise of the teacher sometimes caused didactical problems. The strong focus on lectures and lecture notes meant that the students did not see much value in studying their textbook. If the lecturer had made more references to the textbook during the lectures this might have encouraged students to make greater and more effective use of their textbook. As it turned out, the textbook seemed to be perceived by both the teacher and most students as merely a source of tasks.

During the process of choosing the textbook, the staff focused mostly on clarity of the presentation and on number of tasks. The textbook used in the present study had a large number of different exercises which, if carefully used, could have given the teacher opportunities to individualise the presentations in class. But students worked on mainly drill tasks and they were most interested in the short-term goal of getting correct answers. When they got correct answers they began to believe that they would succeed in mathematics and they did not really need to know the theoretical parts in order to achieve their goals. If the students had been directed to the theoretical sections more often, and if assessment tasks had a stronger theoretical orientation, then this might have persuaded students that it was necessary to read and comprehend the text.

Certainly, the textbook included exercises that were not merely drill tasks. According to the textbook's author, Adams (2006), "other exercises are designed to extend the theory developed in the text" and therefore enhance the students' " understanding of the concepts of calculus" (p. xiii). But the students did not work on these non-drill tasks because they did not see the point of trying to work through the theoretical sections in the book. They thought they did not have time to waste on what they perceived as unhelpful theory. Because so many students were asking for help, the teacher did not refer to the book but usually told them how the task should be solved.

Didactical implications

The findings of the study point to the need for further research into the role of the mathematical textbook at tertiary level. The textbook is intended to be a learning and teaching tool, and, thus, more awareness is needed of student difficulties with the textbook. Perhaps, mathematics education researchers have focused too much, and too narrowly, on learning problems.

There is not the same attention paid to learning theories in the delivery of university mathematics as there is in the teaching of the subject at lower levels. Greater consciousness of factors that should influence the choice of textbook for mathematical courses at tertiary level is necessary. Different textbooks should be evaluated, taking into account different approaches to presentation and treatment of mathematics concepts, students' previous knowledge, and curriculum goals.

In particular, the question of how the textbook should be embedded in the learning and teaching context in order to achieve the goals stated by curriculum needs to be considered. A short account about how the textbook is intended to be used should be given during the first lectures in a semester. More frequent references to the textbook should be made in lectures, especially in relation to showing, explaining, and discussing how important concepts are treated by the textbook author. This could help students to realise the opportunities afforded by the book.

More exercises that encourage use of the book (especially those that require reading the text) should be proposed in the task-solving sessions. Further specific research, probably some *design research* studies, about using of the textbook at tertiary level is needed. It could help to recognise and utilise the potential of the mathematics textbook as a learning and teaching tool.

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Article 3

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Approach to mathematics in textbooks at tertiary level – exploring authors' views about their texts

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The aim of this article is to present and discuss some results from an inquiry into mathematics textbooks authors' visions about their texts and approaches they choose when new concepts are introduced. Authors' responses are discussed in relation to results about students' difficulties with approaching calculus reported by previous research. A questionnaire has been designed and sent to seven authors of the most used calculus textbooks in Norway and four authors have responded. The responses show that the authors mainly view teaching in terms of transmission so they focus mainly on getting the mathematical content correct and 'clear'. The dominant view is that the textbook is intended to help the students to learn by explaining and clarifying. The authors prefer the approach to introduce new concepts based on the traditional way of perceiving mathematics as a system of definitions, examples and exercises. The results of this study may enhance our understanding of the role of the textbook at tertiary level. They may also form a foundation for further research.

Keywords: textbook in mathematics; students' difficulties with calculus; approach to mathematics; research in mathematics education

1. Introduction

To study learning and teaching mathematics at tertiary level is still a young but expanding area of research in mathematics education. Especially calculus and linear algebra have been earlier favoured for research and many results have been obtained. However, only a limited number of studies have been carried out researching textbooks at tertiary level. This study is part of an ongoing larger one considering the role of the mathematics textbook as a learning and teaching tool for first-year students taking the basic calculus course. The results in the main study indicate students have serious problems with meaningful use of the textbook. Many students are not concerned with the theoretical sections in the textbook, they perceive the book as too hard to use and the consequence is that the textbook is reduced to a source of tasks. Randahl and Grevholm [1] suggest that some of students' difficulties might stem from the formal introduction and treatment of the concepts in the textbook. In attempting to shed light on the textbook's role from different

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perspectives, this study aims to explore the authors' thoughts about their texts and about approaches they take when introducing new concepts. There is little knowledge about views and beliefs of the authors who propose the textbooks to the students and to the teachers. Only rarely have some assumptions about the authors' agenda been mentioned in mathematics education research.

Johansson [2, p. 6] writes:

> There exists an author (or a group of authors) and a producer of the textbook, whom one can assume to have the intention to offer a well-made, carefully prepared pedagogical version of a school topic.

Authors' responses might offer some valuable insight in order to better understand the mathematics offered in the textbooks and how it is intended to be learned.

The following research questions are posed:

(1) What characterizes authors' vision of the calculus textbook offered to the first-year students?
(2) What characterizes authors' views about the introduction of new mathematical concepts?
(3) In what ways do these views correspond with the results of previous research?

The character of revisions the authors have done in different issues of their textbooks will also be briefly considered.

2. Theoretical background

2.1. *Calculus and students' difficulties with it*

Calculus is a basic mathematics course at tertiary level. There are mainly two groups of students taking calculus: students majoring in mathematics and students who are taking mathematics as a service subject. The core content of calculus consists of differentiation, integration and differential equations. Some characteristics of calculus as a mathematics subject at the tertiary level are presented by Stewart [3, p. 3]

> Calculus is fundamentally different from the mathematics that you have studied previously. Calculus is less static and more dynamic. It is concerned with change and motion; it deals with quantities that approach other quantities.

The calculus course has traditionally focused on mastery of symbolic methods and applying them to solve problems [4]. This aspect of calculus has also been the emphasis in many textbooks; here from Adams [5]: 'Learning calculus will provide you with many useful tools for analysing problems in numerous fields of interest'.

As mentioned above calculus relies on the concepts of limit and derivative. Although the first-year students are familiar with these concepts some problems might emerge. At the secondary level the focus is often on the intuitive aspect of the concepts, and at the tertiary level the concepts are mostly introduced by formal definition. Many years ago research in mathematics education identified students' serious difficulties with learning calculus [6–8]. Students can often quote the formal definition but have problems making sense of and using it. This problem has been analysed and conceptualized by Tall and Vinner [9] and will be considered later in this article. Also more recent research continues to report students' problems with calculus. White and Mitchelmore's [10] study shows that students are not able to relate and make sense of basic calculus concepts. In the study of Amoah and Laridon

[11], students find it difficult to move freely between different representations of the derivative concept. Sixty percent of the students are not able to estimate the derivative at the point numerically and 74% of the students could not find the derivative at a point from the graph. The study of Viholainen [12] shows students' difficulties with associating formal and informal reasoning. Students avoid using the formal definition in problem-solving situations. The research in mathematics education also tried to provide reasons for students' difficulties. Students' focus on the procedural knowledge without sufficient conceptual understanding and the conflict between students' intuitive ideas with formal definitions have been studied [13]. As mentioned above, calculus is the first course in mathematics at tertiary level. Some of students' possible difficulties might also be embedded in more general problems associated with the transition from the secondary to the tertiary level [14–16].

2.2. *Mathematics in textbooks*

The textbook and lecture notes are the main instructional material that the students traditionally use in work both inside and outside the mathematics classes at the tertiary level. Providing a textbook that enables the first-year students to understand central calculus ideas is a challenging task. Although the calculus textbook is expected, as mentioned in the preface of many textbooks, to be an important learning and teaching tool, the textbook at the tertiary level has not been clearly conceptualized in previous research. The focus on the teacher–student–textbook triangle that has been used in studies regarding the role of the textbook at lower levels cannot be automatically adapted. The role of the teacher at tertiary level is in some ways limited. While at lower levels the teacher is expected to assist the learner in the interaction with the text by preparing the unit from the book and clarifying it [17], the situation at the tertiary level may be quite different. The students may be expected to work with the mathematics in the book on their own. Because of this the decisions made by the authors might have an important impact on the process of learning for those using the textbook. The authors' views about the nature of mathematics and the teaching of mathematics might have consequences for their vision of the mathematics for learning proposed in the textbook. Different views about the nature of mathematics have an influence on how mathematics should be introduced to the learner and how the students are supposed to learn mathematics [18,19]. For example, the Platonic view that perceives mathematics as existing in its own right, and the Aristotelian view, which claims that mathematics is created, lead to different views about teaching and learning mathematics. Assuming the Platonic view, the learner discovers mathematics and because of this mathematics should be presented as a pre-constructed body of knowledge as clearly as possible. The question is: how can pre-construct mathematics be presented to students? The Aristotelian view implies that the learner creates mathematics. The questions then arise: do students have to reconstruct all that has been constructed before? How is, or could this be, possible? Whatever stance is taken, the question about how the new concept should be introduced is highly central in this context. The study of Randahl and Grevholm [1] shows that the introduction of the concept in the analysed calculus textbook has a formal mathematical character without links to practical situations. The definition relies strongly on the concept of limit which is difficult for the students [6,8,20,21]. Many years ago the problem connected with the mathematical definition was conceptualized by Tall and Vinner [9] who introduced the terms *concept definition* and *concept image*. The *concept definition* is defined as the formation of words/symbols used to define a mathematical concept and the *concept image* as 'a total cognitive structure associated with the concept in an individual's mind' [9, p. 152]. The concept image includes all mental pictures, ideas and structures and it develops through experiences. It is important to learn formal mathematics because it is a more powerful tool than the informal, especially when the students are working with proof, but the informal mathematics may also have an important role in the development of the concept image. The concept image has an important influence when the students use mathematics in new contexts.

> When students meet an old concept in a new context, it is a concept image, with all the implicit assumptions abstracted from earlier contexts, which respond to the task. [7]

3. This study

As stated above the purpose of this study is to explore the authors' vision concerning the mathematics textbook proposed to first-year students and their views about the introduction of mathematics concepts in the textbook. A questionnaire has been used as a main method for data collection.

Questionnaire:

(1) What is your vision concerning the calculus textbook?
(2) How do you decide the content of the book? What criteria do you use?
(3) What in your opinion is most important to consider when a new mathematical concept is introduced in the book?
(4) What are, in your opinion, the criteria for 'a good definition'?
(5) When shall a formal definition be introduced?
(6) What preparations are necessary before introduction of formal definition?
(7) What, in your opinion, develops mathematical intuition?
(8) What kind of knowledge, conceptual or procedural, does your book emphasize?
(9) When you write text for the book, who do you see as your prospective reader?
(10) When you write, you use a special style of language. What do you want to achieve when you choose this style?
(11) Could you imagine using other language styles and in that case which?
(12) If you could imagine an ideal student using your book, how would the student respond?
(13) How does this ideal student differ from an average real student, according to your experience?
(14) Who responds most to you as the author concerning the book?

- Students.
- Lectures.
- Other mathematicians.

(15) What kind of revisions did you make in the last edition of your book?
(16) What were the reasons for making these changes?

The questions are mainly embedded in the results of the larger study which show that students have most problems with making sense of the formal definitions and formal treatment of concepts. At the same time the formal definition is frequently used when the new concept is introduced in the textbook. Because of this the questions concerning authors' viewpoints about the role of the definitions in introducing concepts in the textbook are central. The questions are purposely open in order to give the authors an opportunity to reflect upon their views. The first intention was to use the questionnaire with follow-up interviews. The authors were contacted and asked if they were willing to respond to the questionnaire and participate in interviews. All authors expressed their interest for this study which they perceived as interesting and important. Unfortunately, the time factor was a constraint for many of them so the interviews were included. Finally, the questionnaire has been sent to seven active writing authors. They were assured anonymity. Four authors responded to the questionnaire. They represent the Scandinavian countries and the US. All of them are mathematicians and have written a calculus textbook either themselves or as co-authors. They have much experience of teaching calculus courses and three of them are still active university teachers. The responses are presented according to the issues of the vision of the textbook, the introduction of the new concepts, students' understanding of the concepts and the changes made in different issues of the books.

4. Results

4.1. *The vision of the textbook*

The question regarding vision for the mathematics textbook is perceived by one author as important but the most difficult to answer.

> *A1*: This is, without a doubt, the hardest question to answer, and it subsumes many of your other questions.

For the same author the vision of the textbook is a matter of *clarity*, making calculus *interesting*, *simple* and *complete*.

> *A1*: I suppose the single feature of and motivation for my book from the beginning has been "clarity". The other components of my vision for the book have steadily grown over the years. In a nutshell they are to make the treatment of calculus "interesting" and "complete". I think both of these are self-explanatory and require no further amplification at this point.

The term 'clarity' has not been further explained by the author. It is not obvious that 'to be clear' means the same to the author as to the learner. Tall [20, p. 17] warns about the process of *simplification* of complex mathematical idea by 'breaking it into smaller components ready to teach each component in a logical sequence'. The intention is often to make mathematics easier for the students but Tall claims that when the mathematicians see the components as parts of the whole idea, the students may construct the concept image of the components in isolation.

One of the authors has problems with the expression 'vision of the textbook' and prefers to explain 'the reasons for writing the textbook'.

> *A2*: I am not sure that I fully understand the question. But the reason for writing the book was to give students coming directly from high school a hand in the difficult step from school to university.

The transition from secondary to tertiary level is a challenging process for the learner. The procedural approach to calculus at secondary level could lead the students to focus on manipulative rules [22]. The issue of how the textbook is embedded in the problems connected with the process is worth further consideration. Raman [23] conducts an epistemological analysis of how pre-calculus, calculus and analysis texts treated the notion of continuity. She found different epistemological assumptions at each level and concluded that the texts send conflicting messages about status and purpose of the mathematical definition. Also the way of using the textbook on secondary and tertiary level is different. Sosniak and Perlman [24, p. 429] give the following views about the textbook usage at the secondary level:

> The textbook is seldom used as the source for insight into strategies for the solution of the problem or for explanation or for clarification of the concepts underlying the problems students are asked to solve. Occasionally the students report being expected to read the narrative portion of the mathematics text ('the sides of the pages'), but it is true only in a small number of instances. More importantly, perhaps students rarely are encouraged to study this narrative seriously. Instead, they count on teachers to 'explain it correctly'.

In her response about reasons for writing the book author *A2* focuses on the 'volume' of some calculus books and the amount of easy mathematics, which can be problematic for motivation of first-year calculus students.

> *A2*: Equally difficult for the new students was the fact that many calculus textbooks start on a rather low level, which means that we cover hundreds of pages during the first weeks of the course, something which can be utterly frustrating for a new student. Indeed, a student may start the semester, full of intentions to work hard and get a good exam. After a few weeks he is far behind because he tries to read the book, do some problems, and study the examples, all the things we actually want the students to do. Then this student "realizes" that this is not the way to work, and we have actually destroyed his good working habits and good intentions.

The textbook proposed to the students is intended to be used by the average student. Some of the students have a weaker background and perhaps need more extensive exercises. The current groups of first-year students differ much from students in previous years. Artigue [25, p. 484] notices:

> For a very long time, mathematicians have been protected from the problems induced by the democratization of teaching. They are no longer spared. They are more and more faced with students, less culturally adapted, who need in some sense, to learn what thinking mathematically is about.

And Zevenbergen [26, p. 13] says

> In previous times, mathematics has been able to take the elite school leavers who were well prepared for their study of mathematics. However, such backgrounds cannot longer be assumed.

The consequences might be that several students will have problems with learning mathematics and that drop-out-rate will increase. These students probably need a more 'soft' start with calculus.

One of the authors related her vision for the textbook to the idea about different representations of mathematical knowledge:

> *A3*: To show the connection between the four components of mathematical knowledge: 1. formal calculation including proof, 2. geometric interpretations, 3. numeracy and 4. ideas including applications. These are four lines of understanding

> that a student should be able to switch between. They should not be separate lines of thought.

The issue of representations has been considered in the research of mathematics education. According to Lesh et al. [27] different representations are crucial for understanding mathematics concepts. The ability to switch between different representations of the concept to make translations among them is necessary for conceptual understanding of the concept. The students have to not only use but also be able to weigh the advantages and disadvantages of different forms of representation [28]. The study of Amoah and Laridon [11] confirms students' problems with different representations of the derivative concept. Only 26% of the first-year students, after differential calculus course, were able to calculate the derivative of a point graphically. Forty percent of the students were able to estimate the derivative at a point numerically.

Author *A4* related the vision for the textbook mainly to the content of the textbook. According to her:

> There are not many options when the textbook is designed. The core curriculum guides both the content and the order of concepts' presentation.

By proposing the textbook, three of the authors emphasize the necessity to offer 'help' to the learner. In order to achieve this, the textbook shall transmit the knowledge as clearly as possible. Mathematics has to be explained and clarified to the students.

4.2. *The introduction of new concepts*

The authors are aware of the complexity of the calculus concepts

> *A1*: Calculus is sometimes perceived by students as very difficult, especially at the beginning. It isn't that difficult, but it does involve a fundamentally new concept (limit) not present in earlier mathematical studies. This concept is very subtle and only somewhat intuitive.

According to the author the limit concept is fundamentally new for the students. This is not the case for Norwegian students taking the calculus course. The limit concept, as well as the derivative concept, is already introduced in high school.

Again author *A1* emphasizes the 'clarity' of concepts' presentation:

> *A1*: My purpose has always been to describe this, and all the other concepts of calculus that arise from it, as clearly as possible. Only in part does this mean "as easy as possible". I like to say I try to make calculus "as easy as possible but not easier" because attempts to oversimplify actually make things less clear.

All the authors present nearly the same vision about introduction of new concepts. The 'definition approach' is strongly preferred.

> *A1*: Let me say that I consider "definitions" the single most important feature of my and any other mathematics book. If you make the "right" definition of a new concept, the development and use of the concept will proceed smoothly.... It is of utmost importance that it should include EXACTLY what should be included and should exclude EXACTLY what should be excluded.

> *A2*: I guess we all agree on what it takes for a definition to be an important tool in a mathematical theory? And we all agree it takes time and effort to choose the right

definitions in a field. And if they are found, the theory is suddenly much easier to comprehend.

A3: Good definitions, clearly stated theorems with proofs and well-designed examples, applications, and exercises . . . .

A4: Sometimes one has to introduce the concept by examples, just to motivate the students. And after this . . . the definition. All depends on complexity of the concept. But the definition is important.

The usefulness of the definition is mentioned by two authors.

A2: The purpose is of course to make the students realize why it is useful and why it is formulated as it is and what it really says. One should also make a conscious choice when it comes to which aspects of the definition one should choose to include. To tell the whole story may often confuse a reader . . . .

A3: Since mathematics is focused towards calculation, the answer needs to be distinct and useful for calculation. Other qualities of a definition can be described and discussed after it has been done.

In answering the question about when the formal definition shall be introduced the authors say:

A2: Of course that depends on the complexity. Often it is a good thing for a reader to see the definition (or theorem) first, so that one has an idea in advance of where the further text is heading. In other cases the conceptual idea is so hard to catch that one needs some introductory examples.

A3: After typical examples that highlight a certain problem, which the definition addresses.

The question about the style of language the authors prefer to use in their text has been answered as follows:

A1: I always try to write clearly. I never try to "dumb down" my language or act as though I regard the student as less intelligent than I am. I use the first person plural pronoun frequently.

A2: The goal has been to have a very clear language of few words. I have tried to separate the mathematics from applications. The applications included are there mostly to illustrate the many ways the theory can be used. The mathematics is there to understand, to enjoy and to give ideas. It is easier to follow a short explanation than a long one.

A3: A correct style, but to enhance readability it tries to be not too formal. This book has commenting dialogues that are separate from the main text. These dialogues are very informal. They try to illuminate the "struggling dialogue" that often is necessary to find understanding in an abstract and perhaps strange subject.

A4: No comments.

The issue of approach to mathematics has been frequently discussed within the mathematics education community. Traditionally, the mathematics proposed in the textbook at tertiary level is perceived as a formal body of knowledge defined by a sequence of axioms, definitions and theorems. The new concepts are introduced by more or less formal definitions so the results of this study are not very surprising. According to Sfard [29] the working mathematicians are mostly Platonists who presented mathematics as 'an ideal, well-defined body of knowledge'. Over the years many critical voices have been raised concerning the axiomatic – deductive approach to mathematics. Lakatos [30, p. 142] points out some problems connected to the 'deductivist style' of presenting mathematics:

> This style starts with a painstakingly stated list of axioms, lemmas and/or definitions. The axioms and definitions frequently look artificial and mystifyingly complicated. One is never told how these complications arose.

Harel [31, p. 2] addresses this problem to all levels and notices:

> Judging from current textbooks and teaching practices, teachers at all grade levels; including college instructors, tend to view mathematics in terms of subject matter, such as definitions, theorems, proofs, problems and their solutions, and so on, not in terms of the conceptual tools that are necessary to construct such mathematical objects.

Vinner [32, p. 80] discussing the problems connected to the introduction of the concepts by formal definitions concludes:

> This does not mean that the formal definition should not be introduced to the student. However, the teacher or the textbook writer should be aware of the effect that such introduction can have on the students' thinking.

And further he says:

> Thus the role of definition in a given mathematics course should be determined according to the desired educational goals supposed to be achieved with the given students. [32, p. 80].

If and how should the definition be introduced? According to Zandieh [21] the students are able to solve many tasks without using a formal definition of the derivative concept. Also Raman [33] points out that the students do not need the formal mathematics in order to be able to solve problems in calculus. The results of research indicate that students' understanding of concepts is not necessarily informed by that concept's formal definition. Does it mean that it is not necessary to introduce the formal definition? Some other approaches have been proposed. For example, Tall [20] critiques a formal approach using the limit concept definition as the foundation of calculus and he proposes the dynamic approach to calculus by designing the Graphic Calculus software. Gravemeijer and Doorman [34] discussed another approach to calculus based on the idea of *guided reinvention*. It is embedded in Freudenthal's [35] view of mathematics perceived not as a closed and complete system but as an activity and the process of modelling reality. The activity is understood as a *mathematizing* of life subject matters or mathematical subject matter. The latter understanding of the formal mathematics should have foundations in understanding of real phenomena and the instructional design has to create opportunities for emergence of more formal mathematics. The idea is strongly embedded in mathematics learning and teaching in the Netherlands and 'almost all Dutch mathematics textbooks show the impact of Freudenthal's ideas' [36]. First-year students' problems with formal language in mathematics have also been the object of concern within mathematics education. The idea that language should communicate the main ideas of calculus might be difficult to be connected with emphasizing the role of formal definition and symbols.

Dorier and Sierpinska [37, pp. 258–259] refer to the study of Robert and Robinet, and Rogalski about first-year students' difficulties in linear algebra:

> Responding to a questionnaire, students voiced their concern with the excessive use of formalism, the overwhelming number of new definitions and theorems, and the lack of connection with what they already knew in mathematics. It was quite clear that these

students had the feeling of landing on a new planet and were not able to find their way in this new world.

4.3. *Understanding of concepts*

All the authors point out the importance of conceptual understanding. One of the authors deepens it:

A2: Conceptual understanding should be the goal of teaching mathematics. To reach this level of understanding takes good explanations, illustrative examples and an abundance of problems for the student to work with, both easy problems and conceptually demanding ones. Oral explanations, eye to eye, are the most effective ones. In a textbook the explanation should not be too complicated or too long.

The issue of conceptual and procedural understanding has been frequently considered within mathematics education research [38–40]. Understanding of main ideas in calculus should be the foundation for using them in new situations/contexts and also for further learning. The promoting of the conceptual understanding is perceived differently by the authors.

One of the authors relates it to training problems with complete solutions.

A3: To facilitate understanding of the calculation aspect, my book contains many training problems with full solutions – all calculation steps are shown and in addition even commented.

Other author discusses the ways of ‘conveying’ the understanding to the students.

A2: For an author it is important to have a very clear understanding of what the concept means, why it was needed in the first place, and why it is needed now. Next it is important to consider how this understanding can be conveyed to the students. Should one try to explain it carefully in clear text and/or should one try to illustrate the effect in several examples?

The question ‘what develops mathematical intuition’ receives following comments:

A1: Good definitions, clearly stated theorems (and clearly presented proofs of theorems), and well-designed examples, applications, and exercises.

A2: I am not fond of the word intuition. Conceptual understanding should be the goal of teaching mathematics.

A3: What does develop mathematical intuition? The connection to experiences that students are familiar with.

The research showed that mathematical intuition has an important role in learning mathematics. According to Fischbein [41,42] intuition is something that the learner creates and develops when she studies mathematics objects. Results of Pettersson’s [40] study reveal that the students in the mathematics programme expressed their understanding in a formal context in which also intuitive ideas play an important role. Students have to use intuitive ideas and formal reasoning in a dynamic interaction.

4.4. *Changes done in different issues of the textbooks*

In general the calculus textbooks have several editions. In answering the question about response and changes that have been made in different editions the authors say:

A1: In volume, I probably get more responses from the lecturers as reviews are regularly solicited from them by the publisher. However, I do hear directly from students too, sometimes with questions or to point out a (suspected or actual) error, and sometimes just to tell me how much they enjoyed using the book and/or found it opened possibilities for career choices that they had not previously considered. Most mathematicians I have heard from like the book, but some think (incorrectly, I feel) that it might be too difficult for their students.

A:... Only correcting mistakes or making the text slightly clearer or putting in a few extra problems.

A:... the resulting new edition is a considerable improvement, especially in the range of applications considered and the depth of awareness of both the strengths and weaknesses of computer aided calculus it will encourage in students.

A: Over the years I have decided on changes in content (revisions) based on experiences from my own teaching and suggestions from colleagues around the world. While the core content of calculus does not change much, my perception of the best (clearest) way to present it does. Also, the nature of applications that can be made of the core content definitely can and does change over time. In particular, the development of computers and the ability to do symbolic calculations with them (e.g., using Maple or Mathematica) has fundamentally changed the way calculus should be taught.

A: One thing that should be considered is the order in which new concepts are presented. I have occasionally made changes in the order of material to achieve better clarity. Usually, but not always, these have worked. Sometimes it has been necessary to go back to older approaches.

The changes done by two authors in the different issues of their books are closely connected with the growing use of the technology. Two other authors emphasize the range of application and problems. No one relates the changes to the results within research in mathematics education focusing on learning and teaching.

5. Some methodological considerations

The data of this study consist of the responses of only four authors. It might create questions about generalizability. It is important to emphasize that the four books proposed by the authors have been and still are among the most used in calculus courses. The data collection process in this study was somewhat complicated. Many authors had 'time-constraints' because of new projects, new textbooks, etc. This situation is illustrated with an excerpt from personal email correspondence with one of the authors:

And I am very sorry that I will not have time to answer your questions. They are in fact very good thoughtful questions, but such serious questions require good thoughtful answers. There are some profound ideas embedded in your queries and I would need many hours even to give relatively brief responses.

Using the questionnaire as a method to collect the data has both advantages and limitations.

Wellington [43, p. 106] emphasizes the advantages:

... data of this kind [concerning a person or an organization's views] collected by a postal questionnaire may even be richer, perhaps more truthful, than data collected in a face-to-face interview. The respondent may be more articulate in writing or perhaps more willing to divulge views, especially if anonymity is assured.

One of the limitations of using questionnaires is that it is often difficult, if not quite impossible, to ask follow-up questions. For example: when one of the authors talk about 'the right definition' it might be valuable to know more about how she perceives the term. But it was not possible to send follow-up questions, mainly because of the time constraints mentioned above.

Additionally one of the authors was irritated by these questions

> I am working through these questions of yours. Are you sure these are questions to an author of a textbook? I feel as if I am a candidate at some kind of exam. Have I done the right thinking before writing the book? Do I know anything about mathematics didactics? Did I have a purpose, a meaning behind the choices I made? Did I see that I had choices?

In order to avoid using *leading questions* there were no questions related directly to the research in mathematics education. The purpose was also to find out to what degree the authors relate their views to the findings of mathematics education research.

6. Conclusions

This article aims at exploring authors' visions concerning textbooks and introduction of new concepts in the textbook. Considerations about how authors' viewpoints correspond with some existing results in mathematics education are substantial. The analysis of authors' responses indicates the traditional view on the teaching–learning process in which teaching is about the transmission of mathematical knowledge organized as a sequence of well-formulated axioms, definitions and theorems, and learning is best achieved by the student progressing in clearly defined steps. The textbook seems to be conceptualized mainly as a 'transmitter of existing mathematical knowledge'. It fits with what Dreyfus [44] notices that mathematics has often been presented to the students as 'the finished and polished product'. He also warns about the consequences:

> This way of presenting may work well for students who major in mathematics, but it can be difficult for students majoring in science, engineering and taking mathematics as a required service subject. [44, p. 27]

The formal approach to presentation of new concepts is emphasized by all authors. Assuming the complexity of the calculus concepts and students' difficulties all the authors emphasize their attempt to present 'the suitable' definition. Many of the authors of textbooks have been or still are mathematics teachers at tertiary level. But the teachers at tertiary level do not always pay the same attention to theories of learning mathematics as teachers at lower levels in the educational system [26]. Although the authors are aware of students' difficulties, none of them relate the difficulties explicitly to the research within mathematics education. Sfard [29, p. 491] raises the problem of 'a serious conceptual gap' between the mathematicians and researchers in mathematics education.

> On the one hand, there is the paradigm of mathematics itself where there are simple, unquestionable criteria for distinguishing right from wrong and correct from false. On the other hand, there is the paradigm of social sciences where there is no absolute truth any longer; where the idea of objectivity is replaced with the concept of intersubjectivity, and where the question about correctness is replaced by the concern for usefulness.

As mentioned above, one of the greatest challenges in the education at tertiary level is the change in the students' population. In this context the demand is to propose curriculum materials that assist students in the process of learning mathematics. If the problems concerning students' difficulties with learning calculus are more embedded in the discussion about instructional materials, a better approach to calculus might be chosen. This does not mean that the level of the courses should be lower but that the traditional approach to mathematics offered to the students ought to be reconsidered. In the time when some concerned voices are raised concerning the future of the textbook a discussion about the vision for the textbook is needed. The students have to perceive the textbook as valuable and helpful when learning mathematics. The mathematics proposed in the textbook should not only be the evidence of the mathematical knowledge achieved by the mathematics community but also has to take into account how students learn mathematics and how they best make sense of the calculus ideas. The linear view of mathematics with concepts built up in a necessary sequence might be problematic for the first-year students without strong mathematical background. Students who intend to study advanced mathematics, engineering students and science students have different expectations and needs. Emphasizing the necessity of addressing all kinds of readers Tall [45, p. 3] says:

> As an educator, I consider it essential to present the ideas in a sequence that makes sense to students, including those who study the subject for its use in applications without any desire to follow it into more advanced pure mathematics studies.

One of the goals of research in mathematics learning and teaching at tertiary level is 'to improve the understanding of students' difficulties and the dysfunction of the educational system' [46, p. 207]. It might be done by making research results more visible and to make better links between research and practice [46]. Different reforms in calculus were carried out in the past and some changes were noticed in the community of mathematics education. In 1994, Gravemeijer [47, p. 443] optimistically claimed:

> In the community of mathematics educators, the view of mathematics as a system of definitions, rules, principles, and procedures that must be taught as such is being exchanged for the concept of mathematics as a process in which the students must engage.

In 2001, Artigue [46, p. 209] warned: 'Finally, research-based knowledge is not easily transformed into effective educational policies'. This study which has been carried in 2009–2010 identifies that there is still a gap between the mathematicians' view about how the mathematical concepts shall be introduced to the students and what is stated by the results of research in mathematics education. Regarding the textbook as an important teaching–learning tool more consciousness about which ideas about mathematics and about teaching and learning is embedded in the textbook proposed to the learner is needed. Also some empirical studies about the implementation of research results about different approaches to the introduction of new concepts in the textbook at tertiary level might be valuable. The need for more concern about the vision for the textbook at tertiary level as a learning tool is obvious. The traditional way of approaching and treating the main concepts has to be discussed taking into account the findings in mathematical educational research. In the forthcoming paper, the author of this article will take up this challenge herself and propose an alternative design for part of a calculus textbook.

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Article 4

The mathematics textbook at tertiary level as curriculum material - exploring the teacher's decision-making process

Abstract

This paper reports on a study about how the mathematics textbook was perceived and used by the teacher in the context of a calculus part of a basic mathematics course for first-year engineering students. The focus was on the teacher's choices and the use of definitions, examples and exercises in a sequence of lectures introducing the derivative concept. The data were collected during observations of lectures and an interview, and informal talks with the teacher. The introduction and the treatment of the derivative as proposed by the teacher during the lectures were analysed in relation to the results of the content text analysis of the textbook. The teacher's decisions were explored through the lens of intended learning goals for engineering students taking the mathematics course. The results showed that the sequence of concepts and the formal introduction of the derivative as proposed by the textbook were closely followed during the lectures. The examples and tasks offered to the students focussed strongly on procedural knowledge. Although the textbook proposes both examples and exercises that promote conceptual knowledge, these opportunities were not fully utilised during the observed lectures. Possible reasons for the teacher's choices and decisions are discussed.

Keywords: mathematics textbook, lectures, engineering students, procedural and conceptual knowledge, teacher knowledge

1. Introduction

The mathematics textbook has traditionally been perceived as an important resource in the teaching and learning process. However, very little research on the use of textbooks at tertiary level has been conducted. This paper reports a part of a bigger study exploring factors that might influence the role of the mathematics textbook as a learning tool in the context of a mathematics course for first-year engineering students. Research on how the textbook may support students' learning is not straightforward. Tarr, Reys, Reys, Chávez, Shih and Osterlind (2008) claim:

> Despite the dominant role that the mathematics textbook has played, drawing a direct link from the textbook to the students' learning is complicated by other factors, including the teacher's choices and actions, the organization of the school and classroom, and the students' readiness and willingness to learn (p. 248).

Previous research suggests that curriculum materials, and especially textbooks, considerably influence classroom instruction (Eisenmann & Even, 2009). Teaching practice at tertiary level is still a less investigated area within education research. Speer, Smith and Horvath (2010) point out that there is some research *about* teaching practice at tertiary level but not much empirical investigation *of* teaching practice, especially concerning what teachers think and what influences their choices. Although some studies about undergraduate teaching were conducted, for example Weber (2004), Hemmi (2010), Bergqvist and Lithner (2012), yet none of these studies investigated how the mathematics textbook was perceived and used in the teaching practice.

In this study the notion of *teaching practice* is understood as described by Speer, Smith and Horvath (2010):

> Teaching practice concerns teachers' thinking, judgments, and decision-making as they prepare for and teach their class sessions, each involving one or more instructional activities. It includes their planning work prior to classroom teaching, thinking and decision-making during lessons, and their reflections on and evaluations of completed lessons (p. 101).

The textbook is perceived as curriculum material embedded in the *formal – intended – enacted curriculum* model. When conducting the study, the aim was to explore the use of the textbook by the teacher during a sequence of lectures. The main focus was on the following issues: (1) to what extent did the teacher draw on and refer to the textbook when introducing the new concept, and (2) the teacher's choices of, and modifications (if any) on textbook's definitions, examples and exercises.

The model of university teaching in mathematics is dominated by the lecture format and lectures are perceived by students as very important (Bergsten, 2006; Pritchard, 2010). Bergsten notices:

> What the lecturer puts forward is considered (by the students) as the core of the course, the most important issues defining the course, which can be inferred from the common tradition among students to copy and even sell lecture notes, also in cases where there is a textbook available (ibid, p.40).

Previous results (Randahl, 2012) show that the first-year engineering students prefer using their lectures notes rather than their textbook. Thus, investigating to what extent the textbook's approach is adopted by the teacher during the lectures might be valuable when considering the role of the textbook. The mathematics textbook at tertiary level is usually very comprehensive and includes many examples and tasks. In the preface addressed to the teacher, the author of the textbook considered in this study says: "There is a wealth of material here – too much to include in any course. You must select what material to include and what to omit, taking into account the background and needs of your students" (Adams, 2006; p. xiv). Investigating the teachers' decisions regarding the choice of, modification of or omissions of definitions, examples and exercises is important when considering what is offered to the students during the lectures. Additionally the process of how the textbook is chosen for the particular course is also of interest because it may provide some insight into the tertiary level teacher's beliefs about the textbook's role.

The mathematics context of the study was the introduction and early treatment of the derivative concept. The derivative is one of the basic concepts in calculus and is also related to other subjects taken by engineering students, for example, physics.

The questions posed in the paper are:

1. To what extent does the teacher adopt the textbook's approach to introduction and early treatment of the derivative concept during the observed lectures? What modifications, if any, are made?
2. How does the selection of examples and exercises fit with the learning goals for first-year engineering students taking a basic calculus course?
3. How are the learning opportunities/constraints (as pointed out in the content text analysis) of the textbook utilized/overcome during the implementation?

The research questions are explored by using theoretical notions from the curriculum model, learning goals with focus on conceptual and procedural knowledge, teacher knowledge related to the use of the textbook, and results from the main study conducted at the University College. The teacher's decisions are explored through the lens of intended learning goals for first-year engineering students taking the mathematics course. The possible reasons for the teacher's decisions are discussed by using the notions of teacher knowledge and professional identity.

2. Theoretical background

2.1 From formal to enacted curriculum

The curriculum is important for what happens in the classroom. Hiebert and Grouws (2007) note:

> The curriculum the teacher is required to use … surely influences students' opportunity to learn. … The emphasis teachers place on different learning goals and different topics, the expectations for learning that they set, the time they allocate for particular topics, the kinds of tasks they pose … all are part of teaching and all influence the opportunities students have to learn (p. 379).

Regarding the textbook as curriculum material (Love & Pimm, 1996) the study adopted the notions of *formal curriculum*, *intended curriculum* and *enacted curriculum* (Gehrke, Knapp, & Sirotnik, 1992 as cited in Remillard, 2005). *The formal curriculum* refers to the goals and activities outlined by the educational authority or designed in textbooks.

The intended curriculum refers to teacher's interpretation of the written curriculum concerning subject matter content and teaching goals. The teacher chooses the curriculum materials and decides how to use them. *The enacted curriculum* is a transformation of the intended curriculum and refers to what actually takes place in the classroom.

The process from formal to enacted curriculum is complex. According to Remillard (2005):

> Studying the relationship between written curriculum materials and the enacted curriculum necessarily involves understanding teachers' processes of constructing the enacted curriculum, including the role that resources, such as curriculum materials, play in the process (pp. 213-214).

Factors such as a teacher's subject matter knowledge, pedagogical content knowledge, beliefs and goals, identity, perception of students, and perception of curriculum materials may influence her decisions. Ball & Cohen (1996, p. 7) claim that when enacting curriculum, teachers work across five intersecting domains: (1) teachers are influenced by what they think about students, (2) teachers work with own understanding of the curriculum, which shapes their interpretations of what the central ideas are, (3) teachers choose tasks or models from, for example, textbooks in order to design instruction, (4) teachers have to take into account the intellectual and social environment of the class, and (5) teachers are influenced by their views of the broader community and policy contexts in which they work.

Love and Pimm (1996) notice:

> The teacher normally acts as a mediator between the student and the text, and will often provide an exposition of the text and explanations to students in difficulties. This interpretation of the text will be based not only on her construction of the intentions of the author, but on accumulated experience of teaching, including her assessment of what additional explanations the students need to understand the ideas being presented (p. 386).

Stein, Remillard and Smith (2007) identify professional identity as being an important factor that influences the decisions teachers make between the formal and the enacted curriculum. The teachers at tertiary level are usually both teachers and professional mathematicians. As Tall (1991) notices, this fact might influence the way instruction is carried out:

> …a mature mathematician may consider it helpful to present material to students in a way which highlights the logic of the subject. However, a student without the experience of the teacher may find a formal approach initially difficult, a phenomenon which may be viewed by the teacher as a lack of experience or intellect on the part of the student. This is a comforting viewpoint to take, especially when the teacher is part of a mathematical community who shares the mathematical understanding. But it is not realistic in the wider context of the needs of the students. What is essential- for them-is an approach to mathematical knowledge that grows as they grow: a cognitive approach that takes account of the development of their knowledge structure and thinking process (p. 7).

Also Speer, Smith and Horvath (2010) refer to some specific features of the tertiary level teacher's identity and claim:

> ….there are important differences between college and pre-college teachers and teaching: College teachers, for example, are less likely to face limits in their content knowledge. On the other hand, they also have less time with students, making experimenting with new content and activities potentially harder (p. 101).

2.2 Learning goals – the issue of conceptual and procedural knowledge

The question about the kind of mathematics that is necessary for engineering students has been of interest in the mathematics education community for a long time (Kümmerer, 2001). Do the engineering students need the theoretical mathematics or should the focus be more on the procedural aspect of mathematics? At tertiary level, especially when mathematics is perceived as a service subject, the pragmatic approach to curriculum is more common (Petocz & Reid, 2005).

Kümmerer (2001) warns:

> Some traditional courses on higher mathematics are like training camps for computing the value of infinite series, for solving integrals by tricky substitutions, for classifying quadratic forms, for solving ordinary differential equations by completely unexpected transformations, and so on. Everything has to be done against a stopwatch, since these skills are tested in a [timed] final written examination. In such a course 'mathematics' means 'use the right formula at the right time (p. 322).

In Norway, the main goals for engineering education are stated in the document *Rammeplan*[1] by the Ministry of Education. Based on it, each department of University Colleges formulates core curricula with specific subject matter content, learning goals, assessment forms and list of literature. It is expected that the future engineers will acquire sufficient mathematical knowledge and skills to enable them to identify, analyse and resolve engineering problems. The goal that considers ensuring 'solid theoretical foundations' that can be applied to engineering subject matter is strongly emphasized in the core curriculum. The students have to get understanding of the main calculus concepts, for example the derivative, in order to apply them in different engineering contexts.

Hiebert and Carpenter (1992) claim that both conceptual understanding and procedural ability are necessary for success in mathematics. Procedural knowledge allows completing the mathematical tasks efficiently. But when procedural knowledge is not supported by conceptual knowledge, the learners know the rules without knowing why they work as they do. When solving the application problems in calculus the learners have to translate the context situations to the abstract level of mathematics. The identification of the appropriate calculus concepts and utilisation of relationships between them requires conceptual knowledge (Tall, 1991; White & Mitchelmore, 1996). Engineering students have to achieve the procedural fluency, but the conceptual understanding is crucial. Considering the goals for engineering students' learning Jaworski (2009) recognizes the importance of both kind of knowledge by saying: "I want all students to be able to engage with mathematical concepts, to

[1] *Rammeplan*=studyplan

develop both conceptual understanding and procedural fluency and to be able to apply these to their engineering tasks” (p. 1590).

2.3 Introduction and treatment of the concept – the role of definition, examples and exercises

The usual way the textbooks organise the teaching of a new topic is the *exposition-examples-exercises* model (Love & Pimm, 1996). By analyzing mathematics textbooks at upper secondary school and college level, Harel (1987) identified two approaches to the introduction of new concepts: the first one when the computational techniques appear before abstract ideas, frequently used by elementary textbooks, and the second one when abstract ideas appear before computational techniques, more used in advanced textbooks. In the second approach the formal mathematical definition is frequently used.

The role of the definition is explicitly pointed out by some authors of textbooks. For example Adams (2006) says: “As is often the case in mathematics, the most important step in the solution of such a fundamental problem (to find a tangent line to a curve at given point – our comment) is making a suitable definition” (p. 93).

At the same time some epistemological, cognitive and didactical concerns regarding using the formal definition are pointed out in previous research (Vinner, 1991; Raman, 2002). The formal definition might create serious problems in the concept formation and should not be uncritically used by the teachers. Already in 1908 Poincare observed:

> For the philosopher or the scientist, it is the definition which applies to all the objects to be defined, and applies only to them; it is that which satisfies the rules of logic. But in education it is not that; it is one that can be understood by the pupils (as cited in Tall, 1988).

And more recently Tall (2011) notices:

> As a mathematician I seek to develop fully functional mathematical thinking, including precise mathematical definitions and proof. As an educator, I consider it essential to present the ideas in a sequence that makes sense to students, including those who study the subject for its use in applications without any desire to follow it into more advanced pure mathematical studies (p. 2).

Examples play a central role in learning and teaching mathematics (Watson & Mason, 2005; Rowland, 2008; Zodik & Zaslavsky, 2008). Mathematical examples have different nature and may be used with different purpose. Zodik and Zaslavsky (2008) mention two types of examples: examples that highlight some specific features of the concept and examples of how to carry out a specific procedure. Examples can be used for illustrating certain ideas, for example related to calculus, the idea of *average rate of change* and *instantaneous rate of change*. Examining examples (and also non-examples) can help students to understand particular definitions, like for example the definition of vertical tangents. Studying examples should also help students to form a generalization which can be applied in the tasks. However, Love and Pimm (1996) notice that 'many texts do not state directly what generalizations the students are assumed to have made' (p. 387). Studies of how the examples are perceived and used by the students expressed some concerns. Lithner (2003) observed the major role of worked examples, especially when calculus students completed their homework.

Examples may be employed by teachers to help students to build understanding of concepts. Weber, Porter and Housman (2008) describe and discuss three ways that teachers might lead students to consider examples: (1) presenting examples, (2) helping students generate examples and (3) asking students to reason about given examples.

The importance of tasks when learning mathematics is emphasized in previous research. Tasks 'convey messages about what mathematics is and what doing mathematics entails' (NCTM, 1991; p. 24). Tasks also provide the contexts in which students think about subject matter, about what it means to *do mathematics*. Thus, the tasks proposed by the teacher can potentially influence the way students view mathematics and what is really important when learning mathematics. One of the purposes of proposing tasks in the mathematics classroom is to engage students with specific mathematics ideas, concepts and skills. The learner is challenged by non-routine tasks and gets possibility to develop cognitive abilities in order to approach problems in different contexts (Smith & Stein, 1998). Tasks might also have the potential to encourage engagement with the concepts (Bell, 1993) and increase conceptual understanding (White & Michelmore, 2006).

2.4 Teacher knowledge and its relation to the use of the textbook

The mathematics textbook as a curriculum material has a long tradition in teaching mathematics. Earlier research suggests that the ways teachers read, interpret, and use curriculum materials are shaped by their knowledge of and views about mathematics (Graybeal & Stodolsky, 1987; Thompson, 1984). Ball, Thames and Phelps (2008) identify "appraising and adapting the mathematical content of textbooks" as an important part of "mathematical knowledge needed to perform the recurrent tasks of teaching mathematics to students" (p. 400).

Also Remillard and Bryans (2004) note factors as teachers' content knowledge, pedagogical approaches, beliefs and previous experiences as important when discussing the delivery of curriculum. When proposing a conceptual framework for the implementation of mathematical tasks, Henningsen and Stein (1997) emphasize factors as teachers' goals, teachers' knowledge of subject matter and teachers' knowledge of students influencing the process of setting up the tasks in the classroom.

An early characterization of teacher's knowledge is proposed by Shulman (1986). Additionally to *subject matter knowledge* the notion of *pedagogical content knowledge* is introduced. Shulman explains it as follows:

> Within the category of pedagogical content knowledge I include, for the most regularly taught topics in one's subject area, the most useful forms of representation of those ideas, the most powerful analogies, illustrations, examples, explanations, and demonstrations - in a word the ways of representing and formulating the subject that make it comprehensible to others (ibid, p. 9).

And further:

> Pedagogical content knowledge also includes an understanding of what makes the learning of specific topics easy or difficult: the conceptions and preconceptions that students of different ages and backgrounds bring with them to the learning of those most frequently taught topics and lessons (ibid, p. 9).

Based on Shulman's work, Harel (1993) proposes three components of teacher knowledge: (a) knowledge of *mathematics content*, (b) knowledge of *students' learning*, and (c) knowledge of *pedagogy*. Knowledge *of mathematics content* refers to the depth of the mathematics knowledge possessed by the teacher, particularly, their ways of understanding and ways of thinking. Knowledge of *student learning* refers to teachers' understanding of how students learn new mathematical concepts and to awareness of possible learning problems and misconceptions. The considerations of possible learning constraints relate also to the specific group of learners. Knowledge *of pedagogy* refers to teacher's understanding of how to teach in order to facilitate the learning process and achieve the curriculum goals. Factors like learning goals in relation to a specific students group have to be taken into account. The issue of promoting ways of understanding and ways of thinking mathematically is crucial.

With respect to the use of the textbook, the knowledge *of mathematic content* refers to the teacher's deep understanding of mathematics concepts in the textbook and understanding of the ideas behind them. The ability to connect the content knowledge learned in schools (here presented in the textbook) with the teacher's knowledge of mathematics content, is an important feature of the teacher's mathematical awareness (Zaskis, Fraser & Leikin, 2009).

With respect to the use of the textbook the knowledge of student learning refers to teachers' ability to evaluate a textbook's approach to introduction and treatment of the concepts in terms of provided learning opportunities embedded in learning theories and education research results. The critical and careful evaluation should also encompass the nature and cognitive demands of definitions, examples and tasks proposed by the textbook.

With respect to the use of the textbook the knowledge *of pedagogy* refers to choosing/modifying/omitting of definitions, examples and tasks proposed by the textbook in order to implement them in the classroom for a particular group of the students. The learning goals are central. The knowledge of pedagogy refers also to choosing the approach to teaching. Research identifies both teacher-centred approaches and student-centred approaches when teaching at tertiary level (Trigwell, Prosser & Taylor, 1994; Trigwell, Prosser & Waterhouse, 1999).

3. Methodology

3.1 The setting of the study

This case study was conducted at a University College in Norway in the context of the basic calculus course for first-year engineering students. The course was compulsory and covered topics from differential and integral calculus such as functions, limits, derivative, and applications of derivatives, integration, and differential equations. Approximately 100 students were enrolled in the course. The textbook used during the course was *Calculus. A complete course* by Adams (2006). The textbook is used around the world and is a popular one in the Nordic countries. The author claims: "The text is designed for general calculus courses, especially those for science and engineering students" (p. xv). No teacher manual is available on how to use the textbook. The teacher who was responsible for this course was an experienced lecturer who was respected by the students. According to the students the teacher was always well prepared for the lectures and was helpful when they had any mathematical problems. The students got a detailed syllabus, indicating topics and sections at the beginning of the semester. So they had the possibility to read the relevant text in the textbook in advance. The students took their own lecture notes.

3.2 Learning opportunities offered by the textbook used in this study

Regarding the textbook as an important factor in students' learning process, the issue of what learning opportunities the textbook offers was found interesting to be investigated. A text content analysis of some sections of the Calculus by Adams (2006) was conducted.
The learning opportunities offered by the textbook were considered by focusing on the nature of definitions, the kind of context when introducing new concepts and problems and emphasis on conceptual and procedural knowledge. This section is a brief review of that investigation and its results (Randahl & Grevholm, 2010). The reason for including it is that the categories used in the text content analysis will be used in the analysis of the lecture observations in this

study (section 3.4).

The first three sections; *2.1. Tangent Lines and Their Slopes*, *2.2 The Derivative* and *2.3 Differentiation Rules* of the chapter "Differentiation" were analysed. In every section, the introduction and treatment of the concept, definitions, examples, and exercises were examined. The concepts of formal and informal definition were used when considering the introduction of the derivative. A formal definition was defined as the concept definition accepted by the mathematical community (Tall & Vinner, 1981). An informal definition was defined as the verbal explanation of the concept without using mathematical symbols.

In the analysis of the examples the justification aspect and new context aspect were studied and the following categories were used:

1. Worked examples (only explicit solutions that can be used directly to find correct answers when working with exercises; no focus on justification).
2. Examples intended to increase understanding of the concept (by using different contexts or where justification is required).

The exercises were categorized according to their emphasis on procedural or conceptual knowledge. The following categories were used:

1. Exercises that mainly require the use of particular procedures.
2. Exercises that require some conceptual preparation before one can use a procedure.
3. Exercises requiring justification of the solution or in which a new context is used.

The results showed that the introduction of the derivative concept in the textbook is formal and relies strongly on the limit concept. The definition of the derivative is given as the limit of the Newton (i.e., difference) quotient. There is no practical context or situation that explains the necessity of extending the existing student knowledge. Taking into account students' poor previous knowledge (Randahl, 2012), the textbook's formal introduction of the derivative was considered as a constraint when students' should make sense of the concept.

The majority of the total of 22 examples provided in the sections was categorized as worked examples. They mainly present particular procedures in order to solve problems. Only one worked example points out two different ways to solve a given problem.

Among the total 140 examined exercises, 76 exercises (54%) were placed in the first category and only 21 exercises were considered as requiring some justification. There are no exercises that require explanation of the meaning of the derivative concept. There is rarely a focus on situations in which a function fails to be differentiable.

It was noticed that the set of exercises is graded in a particular way. The exercises that require mostly knowledge of easy procedures for obtaining correct answers are placed in the beginning of each exercise set. Exercises with emphasis on conceptual knowledge are placed later. This might create difficulties for students who had problems with effective use of the textbook and who never might reach to work with the tasks that focus on conceptual knowledge. One of the conclusions was that some assistance from the teacher might be necessary when students attempted to solve the exercises.

3.3 Data collection

The teacher was interviewed in the beginning of the semester. The focus was on the process of evaluation and choice of the textbook for the particular course, together with the teacher's experiences of how the textbook was perceived and used by the students**.**

The data was obtained during three weeks of the semester when the students were working with the derivative concept. A sequence of six lectures was observed and the field notes were collected. The observations focused on: to what extent did the teacher rely on the textbook, how often the teacher referred to the textbook, what examples and exercises were chosen/omitted, and what modifications, if any, were done. This data was complemented with information obtained through many informal talks with both the teacher and the students. The conversations usually took place in the breaks between and after the lectures. The questions asked by the researcher were connected to the lecture observations for which some additional explanations were needed. For example the students were asked about their reasons for looking (if observed) in their textbooks during the lectures. Similarly the teacher was asked about the reasons for using/not using a particular example or exercise from the textbook.

3.4 Data analysis

The field notes taken during the lectures were quantitatively and qualitatively compared with corresponding sections in the textbook. The qualitative part was mostly about counting how many examples and exercises proposed by the teacher during the lectures were collected from the textbook. The introduction of the derivative proposed by the teacher during the lecture was analysed with focus on the nature of, if any, modifications of the textbook's approach, the context used, and the kind of definition. Each example and exercise proposed by the teacher was first categorized as (1) from the textbook or (2) from other sources. Further, each example and exercise was examined using the categories from text content analysis (section 3.2). When analysing the examples and exercises the focus was on the choices and modifications, if any, made in order to promote conceptual knowledge.

Three types of modifications, as proposed in the study of Lee, Lee and Park (2013) were considered: context modification, condition modification, and question modification. Context modification refers to changing the context of tasks or making them student-friendly. Condition modification refers to adding, deleting, or transforming the conditions in tasks, also by using "what-if-not" strategies. Question modification refers to changing what students are required to answer. Also changing the tasks into a more open-form or demanding some investigation are examples of question-modification (Crespo, 2003; as cited in Lee et al., 2013). Finally the focus was on textbook examples and exercises that were omitted; what categories were dominating among them. The omitted examples and exercises were examined using the categories from text content analysis. The reasons for the teacher's decisions were discussed during the informal talks with the teacher.

4. Results

4.1 Choosing the textbook for the study

This section is not a part of the data analysis provided in the study. The reason for including it is that the information obtained during the interview with the teacher illuminates the process of evaluating and choosing a textbook for a particular course at tertiary level. It contributes to the explanation of the teacher's approach to the textbook and thereby provides a connection to the research aim.

The interview and informal talks with the teacher showed that there was not an established procedure of evaluating and choosing the textbook for a particular course. The main reason for the latest change of textbook was students' complaints about difficulties with understanding the previous textbook.

When explaining about the process of choosing the textbook the teacher said:

> It seems that the textbook is always difficult for them. They complained about ……a lot of things, for example the formal language. It was the reason why we proposed another textbook last year. I looked in some calculus textbooks. But it was not easy….you do not know them before you use them in the class. Among them was Edward and Penny's book. This one I know from the time I went to school myself. And Thomas' textbook and the few others, I cannot remember the names of these. It was Adams and Edward and Penny I looked most at. The problem with Edward and Penny is that it contains so huge amount of tasks, text and images, and so on. I was afraid that the book will be perceived as too comprehensive.
>
> Later I talked to some other colleagues in Sweden and one of them recommended strongly 'Calculus' of Adams. The book seemed to be well arranged and offered a lot of different tasks to engage students.

Further the teacher emphasised the role of the lecture notes:

> Most students have this experience that you are doing actually well without the book as long as you follow the lectures. And it is true, I think. Lecture notes are used very much. Most students have to prioritize the time. So they prioritize the lecture above all, really.

It shows that the process of evaluation and choosing of a textbook was not established in the department. Student complaints about formal language was a reason given for the change. When talking about the role of the textbook and the lecture notes, the teacher believed that most students ignored the textbook and focused on the lecture notes.

4.2 The lecture design

The sequence of six lectures regarding the introduction and early treatment of derivative concept were observed. Each of the observed lectures had the form of a presentation of a preannounced topic. The students wrote their own notes. The lectures were structured in the traditional way. First the concept was defined, and then the teacher solved some examples and exercises on the blackboard. There were the sections during the lectures when the students worked by themselves with tasks proposed by the teacher. The teacher usually went around and answered students' questions. If a particular task led to difficulties, the teacher solved it on the blackboard.

Here is the excerpt from the informal talk with the teacher about her preparation to the lectures:

I (Interviewer): How do you usually prepare your lectures?
T (Teacher): I follow the textbook suggestions in respect to content. But I have to make it understandable for students. So as many as possible understand what it is all about ....also the mathematics.
I: To what extent is your preparation based on the textbook?
T: As I said, the content, also the topics in the textbook….. But I use also other books, for example Calculus by Thomas. You know, you find different useful things in different textbooks. For instance other examples which are more interesting for students…
I: What do you mean by 'more interesting'?
T: Primarily I think about context. Not every student is interested in mathematics, so I try to motivate them…find interesting context, more relevant for engineers.

We notice that the teacher emphasizes understanding and wants to motivate the students and make them interested in mathematics.

4.3 Introduction of the concept

While introducing the concept of the derivative, the teacher closely adopted the textbook's approach. The following definitions, all from the textbook, were presented on the blackboard and explained:

- the definition of the tangent line
- the definition of the slope to the tangent line
- the Newton (i.e., difference) quotient
- the slope of the curve at a particular point and the slope of the normal.

Finally the definition of the derivative was presented as the limit of the Newton quotient as follows:

The derivative of a function f is another function f' defined by

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

at all points x for which the limit exists (i.e., is a finite real number). If $f'(x)$ exists, we say that f is differentiable at x.

The definition is exactly as the one proposed in the course textbook (Adams, 2006; p. 98).
In the following the teacher presented different notations for the derivative, Leibniz notations $\frac{dy}{dx}$ and $\frac{d}{dx}\left(f(x)\right)$ were introduced. When explaining these alternative notations, the teacher referred to the particular pages in the textbook (ibid, pp. 102-104).
In the informal talk I asked the teacher about the role of the definition in the calculus course. According to her, the formal definition was not so important for engineering students.
She said:

I usually show 1-2 examples about how to use the definition in order to get some formulas. Otherwise the students may find more in the book…. especially those who are more interested in mathematics.

To sum up, when the concept of derivative was introduced, the sequence of definitions was followed as proposed in the book. No discussion of the ideas behind the derivative concept was offered. No modifications of the formal character of the definitions were observed. The presentation and explanations of the notions were mathematically correct. The observations indicate that the teacher has solid knowledge *of the mathematics content* (Harel, 1993).

4.4 Examples proposed during the lectures

The examples were presented and solved on the blackboard. The teacher was methodical and careful, and explained in great detail. There was no discussion and no questions were posed to the students. Of the seven examples proposed by the teacher, six examples (86%) were from the textbook of Adams. One example was picked from other calculus textbook. According to the categories from the text content analysis, the six examples (from Adam's textbook) proposed during the lectures may be distributed as shown in the table below.

Table: Distribution of examples by categories

Worked examples	*Examples that intend to increase understanding of the concept*
Finding the equation of the tangent line to a particular curve at a particular point	Finding equations of any lines that pass through a specific point and are tangent to the curve given by a specific equation
Finding the slope of a particular curve at a particular point	
Using the definition of derivative to calculate the derivatives of two functions, (one linear and one quadratic function)	
Using of differentiation rules for sums and differences	
Using the product rule and the quotient rule	

Five of the textbook's examples (83%) were from the category *Worked examples*, one example was from the category *Examples that intend to increase understanding of the concept.* It shows that the teacher, when proposing examples, emphasised the procedural aspect of mathematics. One of the examples from the textbook that was not proposed by the teacher during the lectures was the following:

Does the graph of $y = |x|$ *have a tangent line at* $x = 0$*?* (Adams, 2006; p. 96)

According to the categories used in content text analysis, this was an example that potentially could have increased understanding of the tangent concept. It points out cases when graphs have tangent lines everywhere except in some points, here the origin. During the informal talk, the teacher explained that she considered the absolute value function to be a difficult issue for the students. It was based on the experiences that students generally had problems with grasping the idea of absolute value. Because of this, the example was initially omitted, but the absolute value concept should be taken later during the course.

The example that was selected from another calculus textbook was as follows:

How fast does the surface area of a circular disk increase in relation to the radius r at the moment when r = 2?

The example provided the idea of the derivative in a technological context. Considering this example in relation to the categories used, it was one which definitely aimed to increase students' understanding of the derivative concept. The students have to identify the derivative concept embedded in the context. Here the understanding of the connection between the different variables was necessary in order to state the equation and use the appropriate procedure. These types of tasks have potential to increase conceptual understanding (White & Michelmore, 2006).

From the informal talk with the teacher after the lecture:

I: I wonder how you choose the examples for the lecture.

T: Usually I consider each example from the textbook. First if it is useful, second if it is not too difficult for the students and so on….. Some of them I used at the lectures but I also review examples from other books.

I: What is the most important factor when you consider the example for the students?

T: Actually…. it is the context. I think it is very important to motivate students to learn mathematics…. They have to see that it is useful for them as future engineers. In the introduction of the derivative is not so easy to find examples or tasks with [appropriate] context. But later in the semester it is quite crucial. Especially when working with differential equation.

To sum up, the majority of the examples proposed by the teacher were from the category worked examples. They were clearly mathematical examples and focussed mainly on how to carry out a specific procedure (Zodik & Zaslavsky, 2008). No modification of the nature of chosen examples from the textbook was observed. An example with a context was picked from another calculus textbook and offered to the students. This example gave the chance to increase conceptual understanding. The teacher's explanations about the reason for omitting the 'absolute value' example indicated that she made some evaluation of what the textbook offered. The evaluation was related to previous experiences (Remillard & Bryans, 2004). Regarding the ways the teacher led the students to consider examples, it was clearly a form of presentation (Weber, Porter & Housman, 2008). The students were not encouraged to work actively with the examples.

4.5 Exercises proposed during the lectures

After the teacher's solving of examples, the students started to work with exercises. The teacher wrote a list of exercise numbers on the blackboard. The students could discuss with

each other on how to solve the exercises. The teacher walked around and helped if needed. Some of the solutions were discussed on the blackboard.
Here are some examples of what the exercises were about:

- Find equations for the tangent lines to the particular curves at the point indicated
- Calculate the derivative of the given function directly from the definition of derivative
- Calculate derivatives of particular functions using the differentiation rules.

The exercises were purely mathematical. The majority was from the category *exercises that mainly require the use of particular procedure* as described in the content text analysis of the textbook. Althought some of the exercises proposed by the teacher required more complicated calculations, they could be solved by using a procedure described in one of the examples. It fits with the results of previous research (Lithner, 2003).
One of the exercises was from the category *exercises which require some conceptual preparation before one can use a procedure*:

Find the coordinates of points on the curve $y = \frac{x+1}{x+2}$ *where the tangent line is parallel to the line* $y = 4x$.

Here the fact that the tangent line should be parallel to the given line has to be considered before some obvious procedures might be used.
None of the exercises proposed during the observed lectures required any direct justification. There were no observed modifications of the textbook's exercises. No extra context or questions were added. The answers obtained by using a procedure were not asked to be additionally justified.

4.5.1 Some of the omitted exercises

In the following, two textbook's exercises that were omitted during the lectures are presented:

Exercise 1
Use a graphic utility with differentiation capabilities to plot the graphs of the following functions and their derivatives. Observe the relationships between the graph of y and y' in each case. What features of the graph of y can you infer from the graph of y'?

a) $y = 3x - x^2 - 1$ *b)* $y = x^3 - 3x^2 + 2x + 1$ *c)* $y = |x^3 - x|$ (Adams, 2006; p. 105)

Here the relationship between the function and its derivative should be recognized and discussed. Previous research showed that students had serious problems with similar problems. In the study of Amoah and Laridon (2004) the students were given a diagram in which a particular curve was indicated as the derivative of the function. They should identify the graph of the function from a collection of curves in the diagram and explain their choice. 57% of students did not identify the graph of the function correctly from the graph of its derivate. Only 9% of the students got the correct answer and explained their choice.

Exercise 2

The volume of water in a tank t min after it starts draining is

$$V(t) = 350(20 - t)^2 L$$

(a) *How fast is the water draining out after 5 min? after 15 min?*

(b) *What is the average rate at which water is draining out during the time interval from 5 to 15 min?* (Adams, 2006; p. 136)

This exercise might be especially valuable for engineering students. It proposes a potentially interesting context and the solution requires understanding of the main idea of the derivative.

To sum up, although the teacher emphasized the importance of context, the tasks proposed during the observed lectures were without a context. The exercises focused strongly on procedural skills. The choice of exercises indicates that the teacher regarded procedural fluency as an important goal for learning.

It fits with the teacher's view expressed during an informal talk:

I: What is most important when you chose the exercises for lectures?

T: I think about many things. They have to be varied and they should preferably be located in an engineering context, if possible. The fluency in skills is very important. The students have to be very good in using procedures. There are many interesting and really good examples and tasks in the textbook. But there is not much time to work with them. So I have to make some choices.

5. Discussion

In this section the results of the investigation are discussed. The research questions posed in the study are briefly answered. The possible reasons for teacher's choices during the textbook implementation are indicated.

The case study presented in this paper investigated the teacher's use of the textbook during a sequence of lectures for first-year engineering students. Perceiving the textbook as a formal curriculum material, the way the teacher enacted the textbook during the lectures, teacher's decisions and possible reasons for making those decisions were of main interest in the investigation. The teacher's decisions were explored through the lens of intended learning goals for the students.

The first research question focused on the extent the teacher adopted the textbook's approach to introduction and early treatment of the derivative concept. Previous research about the use of the textbook at lower levels has shown that teachers rely on the text when teaching mathematics (Johansson, 2006). The results of the present study suggest that a similar situation might also be the case at tertiary level. The teacher in this study seemed to perceive the textbook as an important knowledge source. The observations showed that the textbook clearly guided the instruction. The textbook's approach to the introduction of the derivative concept was adopted and the majority of definitions, examples and exercises proposed during the lectures were chosen from the textbook. The teacher followed closely the textbook's definitions and the formal approach without any obvious modifications. When introducing the concept the teacher adopted the linear view of mathematical knowledge as proposed in the textbook. According to Love and Pimm (1996) this way of presenting mathematics is based on the idea of sequentiality of learning. But the linear sequence of mathematics presented in the textbooks creates some concerns within educational community. Leapfrogs (1975, p. 4) warns: "….learning is not necessarily made any easier if some carefully planned programme is followed". And further: "The idea (of sequentiality) rests upon the assumption that when something has been taught that it has been learned" (as cited in Love & Pimm, ibid; p. 384).

The second research question focused on how the selection of examples and exercises fit with the learning goals for engineering students. The decisions regarding the choice of definitions, examples and tasks related to the teacher's interpretation of the formal curriculum and the role of curriculum. Different factors, like teacher's knowledge, beliefs, goals and identity might influence the decisions and choices (Remillard, 2005; Ball & Cohen, 1996). As stated above, the goal for engineering students when learning mathematics should be to achieve the conceptual and procedural knowledge in order to solve engineering problems. To obtain procedural fluency with understanding of fundamental ideas of the concepts is emphasised in the core curriculum. It seemed that the teacher perceived the fluency in procedural knowledge as important learning goal. The chosen sample of examples and exercises as observed during the lectures was clearly promoting procedural knowledge. Only few examples and exercises focused on conceptual understanding. The informal talks indicated the teacher's beliefs that procedural fluency was especially important for engineering students. It is in agreement with the literature on mathematics courses for engineering students (for example Kümmerer, 2001 as cited in section 2.2).

The third research question focussed on the way the learning opportunities/constraints of the textbook were utilized/overcome during the implementation. According to the results of the content text analysis of the textbook, the formal approach was considered as a possible constraint when learning mathematics (Randahl & Grevholm, 2010). The approach used during the observed lectures did not overcome this constraint. Introduction of the new concepts given in the textbook, in which the abstract idea of the concept appears before computational techniques, was adopted. Many students entering the basic calculus course have poor ideas about the main concepts (Randahl, 2012; White and Mitchelmore, 2006), and because of this, they might perceive formal definitions as difficult. Results from previous research have shown that students have difficulties making sense of the definition of the concept of derivative and using it in problem solving situations (Orton, 1983; White & Mitchelmore, 1996; Viholainen, 2008). Randahl and Grevholm (2010) also emphasised that better explanation of the necessity of an introduction of the derivative could contribute to improve the students' motivation. During the introduction sequence of the derivative concept it might be essential to give the students an overview of the main ideas of calculus and focus on the connections between them (Tall, 2011). The observed sequence of lectures did not offer any discussion about the idea behind the derivative. To get understanding of the idea behind mathematics concepts is emphasized as essential part of the learning process (Apostol, 1967; Selden & Selden, 2001). Although some examples about using the definition in obtaining the basic formulas were shown during the observed lectures, the meaning of using the formal definition was not clearly emphasised. Fishbein (1994) states that presentation of the concept by a formal definition is useful only if the definition will be used actively by the students. Although the observations indicated that when following closely the formal definition approach to the concept, the teacher was aware of the cognitive demands of such an approach. A very comprehensive explanation of the definition and of examples was given, probably in order to help students to achieve an understanding of the concept. The observations showed that the potential of the examples and exercises in order to promote conceptual knowledge was not fully utilized during the observed lectures. Tasks which focus on procedural knowledge are important because working with them affords students opportunity to achieve procedural fluency which is important for engineering contexts. However, procedural knowledge alone does not form the essence of mathematics (Eisenhart, Borko, Underhill, Brown, Jones & Agard; 1993). Students with only procedural knowledge can obtain correct answers when working with problems, but they do not understand why they use a specific procedure (Hiebert & Lefevre, 1986). The informal talks indicated that the teacher believed that the context was important for engineering students in order to increase their motivation. Only one example (from another calculus textbook) proposed during the lectures was situated in a context potentially interesting for engineering students. The choice of the example from another calculus textbook showed that the teacher attempted to propose examples/tasks that she believed was important. However, no attempt for modifications of textbook's examples and tasks during the lectures was observed.

The choice of examples and tasks indicated that the teacher in the present research might not be aware of which problems students usually have when studying the derivative. For example, tasks related to graphical understanding of the derivative are crucial when studying calculus (Asiala, Cottrill, Dubinsky & Schwingendorf; 1997). To understand the derivative as a function, it is important to study the relationship between the derivative of a function at a point and the slope of the line tangent to the graph of the function at that point (Asiala et al., ibid). Both the textbook's examples and the tasks considering graphical understanding of the derivative were omitted by the teacher. Not offering exercises that emphasise important features of the concept to the learner might indicate some lack of pedagogical content knowledge (Shulman, 1986; Harel, 1993). More insight into the research results about students' learning could provide some guidance for the teacher when choosing examples and exercises from the textbook.

When considering the value of the current paper, the following question appears: How may this case study contribute to improve the teaching of mathematics at tertiary level when focusing on the use of the textbook? When discussing how the textbook was used by the teacher, it is important to notice that the teaching practice was observed only during a short time of the semester. More research concerning how teaching practice develops or maybe changes over time could be valuable. It is important to focus on the issue of the pedagogical awareness (Mason, 2002; Nardi, Jaworski & Hegedus; 2005) when considering the teaching practice at tertiary level. The teacher in this study was an excellent mathematician with high level of subject knowledge. However, the study showed that even when the textbook offers examples and tasks of conceptual nature, these examples and tasks were not being used by the teacher. Some of the examples and tasks that promote procedural knowledge could also, by using some modifications, provide opportunities for students to achieve conceptual understanding. This potential was not utilized during the observed lectures. Some of the choices and decisions might indicate lack of knowledge when appraising and adapting the content of the textbook (Ball et al., 2008).

The results of this study provide an argument for the necessity of more investigation and discussion about the way the textbook is conceptualized and used in undergraduate mathematical courses. Existing research at tertiary level focussed on the quality and the role of the lectures (Bergsten, 2006; Pritchard, 2009; Weber, 2004) without paying much attention to how the textbook influenced what was offered to the students during the lectures.
The study offers also some insight into the mathematics teaching culture at tertiary level. When considering the how the textbook was used by the teacher during the observed lectures, the issue of some traditional format of instruction at tertiary level might be significant. The tendency to present mathematics to the students as a polished product has been pointed out in literature (Davis & Hersh, 1981; Alsina, 2001; Weber, 2004). This might influence the way the textbook is perceived and used in the teaching practice at tertiary level. Although the study did not directly focus on learning styles, the relation between the approach to teaching and the way the textbook was used might be worth to be investigated in the future. The teacher-oriented approach to teaching might be a factor that promotes 'direct transmission' of the textbook's presentation and treatment of the derivative.

It could be also interesting to focus on other factors that possibly influence the role of the textbook in teaching practice. Apple (2010) claims that the conditions that teachers face and the political economy of text publishing influence the perception and the use of the textbook by the teachers. Also Borg and Gall (1989) notice: "Teacher behaviour in the classroom is affected by what happens in the broader setting such as the department, …, and these systems must be taken into account when studying the local scene" (p. 407). All the mentioned factors might influence a teacher's decisions and should be focused upon in future research about the use of the textbook in an institutional setting.

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Appendix B

SPØRRESKJEMA

Navn ____

Dato ________

Spørsmål 1

Forklar, så godt du kan, hva mener vi med en *funksjon*.

Spørsmål 2

Finn grenseverdiene:

a) $\lim_{x\to 4} 2x+3 =$

b) $\lim_{x\to 0} \dfrac{x^2-2x}{3x} =$

c) $\lim_{x\to 0}(1+x)^{\frac{2}{x}} =$

Spørsmål 3

Deriver funksjonene:

a) $f(x) = 2x^3+5$ $\qquad f'(x) =$

b) $f(x) = 3\sin 2x$ $\qquad f'(x) =$

c) $f(x) = 2\sqrt{3x}-5x$ $\qquad f'(x) =$

Spørsmål 4

Gitt funksjonen $f(x) = x^2+3$. Den deriverte $f'(x)$ til $f(x)$ er lik 2x.
Forklar hva den deriverte $f'(x)$ forteller om selve f - funksjonen.
Vi kan finne verdien av den deriverte for en konkret x; for eksempel for $x=4$ får vi
$f'(4) = 2\cdot 4 = 8$
Hva forteller tallet 8 oss?

Spørsmål 5

I den videregående skole ble du kjent med den mer ”formelle” definisjonen av den deriverte.
Vi repeterer den her:

$$f'(x) = \lim_{\Delta x\to 0}\frac{f(x+\Delta x)-f(x)}{\Delta x}.$$

Forklar hvordan du forstår denne definisjonen. Bruk den deretter til å vise at $(x^2)' = 2x$.

Spørsmål 6

a) Kan du gi en forklaring på hva du mener du kan <u>bruke</u> den deriverte til?

b) En sykkel er utstyrt med fartsmåler som måler farten $v(t)$ ved tidspunkt t.
Hva mener du er den fysiske betydningen av størrelsene

(1) $\dfrac{v(t_2)-v(t_1)}{t_2-t_1}$ og (2) $\lim\limits_{\Delta t\to 0}\dfrac{v(t+\Delta t)-v(t)}{\Delta t}$?

Du kan sette (1) og (2) i alternativene nedenfor:

☐ gjennomsnittlig fart

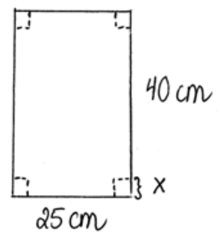
☐ momentan fart

☐ gjennomsnittlig akselerasjon

☐ momentan akselerasjon

Spørsmål 7

En rektangulær plate har sidene 25 cm og 40 cm. Av denne platen ønsker vi å lage en eske ved at vi klipper bort et kvadrat av hvert hjørne og bretter opp sideflatene. Kall kvadratets side for x; se figur

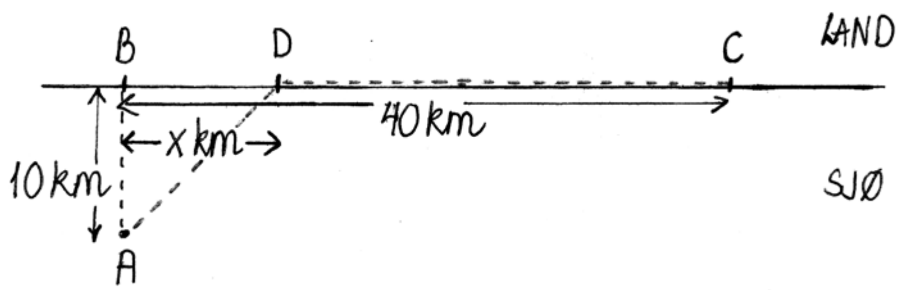


a) Vis at eskens volum kan skrives
$V = 4x^3 - 130x^2 + 1000x$

b) Finn det <u>største</u> volumet esken kan få.

Spørsmål 8

Fra en oljeplattform i punkt A skal det legges en rørledning til mottaksterminalen for olje som ligger i punkt C; se figur



Det er oppgitt at avstand AB er 10 km og avstand BC er 40 km.

Det er vanskeligere å legge rørledning i sjøen, derfor klarer man bare å legge 1 km per dag der. På land går det raskere og man kan legge 2 km ledning per dag.
Anta at ledningen går fra A via punkt D til C. Kall avstanden BD for x km.
Prøv å finne en funksjon $t(x)$ som angir tiden det tar å legge rørledningen fra A til C.
Hvordan kan du bestemme hvor punkt D bør være for at leggingen av rør skal gå raskest mulig?

Spørsmål 9

Hvilke av oppgavene 1 – 8 foran syns du var lette å svare på og hvorfor?
Hvilke syns du var vanskelige og hvorfor?

Spørsmål 10

Hvorfor mener du at vi må ha matematikk i ingeniørutdanningen? Kryss av for de 2 viktigste grunnene:

Fordi:

- ☐ det er interessant
- ☐ det er bruk for matematikk i andre fag
- ☐ man får bedre forståelse for matematikkens rolle i hverdagen/arbeidslivet
- ☐ matematikk utvikler den logiske tankegangen
- ☐ det er viktig å bli flinkere til å regne
- ☐ annet;

hva ____

____

Spørsmål 11

Hvordan oppfatter du matematikk som fag:

- ☐ Meget vanskelig
- ☐ Vanskelig
- ☐ Noe vanskelig / noe lett
- ☐ Lett
- ☐ Meget lett
- ☐ Ingen mening

Kommentar:____

____

Spørsmål 12

Hva syns du er viktigst når du lærer matematikk:

- ☐ Forståelse
- ☐ Interesse
- ☐ Se hvordan matematikk brukes; anvendelse
- ☐ Få riktig svar på oppgaver
- ☐ Formler og metoder
- ☐ Lære å sette opp tekstproblemer på matematisk form slik at de kan løses
- ☐ Andre grunner;

hvilke? ____

____

Spørsmål 13

Hvilken av følgende arbeidsmåter *tror du* fungerer best for deg når du arbeider med matematikk:

- ☐ Følge forelesninger
- ☐ Jobbe mest alene
- ☐ Gruppearbeid
- ☐ Annet;

hva: ____

____

Spørsmål 14

Hva tror du blir de viktigste læringskildene for deg i læringsprosessen?

Kryss av de 2 viktigste:

- ☐ Lærebok
- ☐ Forelesningsnotater
- ☐ Hjelp fra lærer
- ☐ Samarbeid/diskusjon med medstudenter
- ☐ Annet;

hva: ____

____

Spørsmål 15

Hvilke forventninger har du til dette Kalkulus-kurset?

- ☐ Stå til eksamen
- ☐ Gjøre det bra til eksamen
- ☐ Få bedre forståelse for anvendelser/bruk av matematikk
- ☐ Bli flinkere til å bruke formler og metoder
- ☐ Få mer interesse for matematikk
- ☐ Annet;

hva: ____

____

Takk for alt du svarte!

Questionnaire

Name ____

Date _________

Question 1

Explain as well as you can how you understand the concept of function.

Question 2

Find the limits:

a) $\lim_{x \to 4} 2x + 3 =$

b) $\lim_{x \to 0} \frac{x^2 - 2x}{3x} =$

c) $\lim_{x \to 0} (1 + x)^{\frac{2}{x}} =$

Question 3

Find the derivatives of the given functions:

a) $f(x) = 2x^3 + 5$ $\qquad f'(x) =$

b) $f(x) = 3\sin 2x$ $\qquad f'(x) =$

c) $f(x) = 2\sqrt{3x} - 5x$ $\qquad f'(x) =$

Question 4

Let $f(x) = x^2 + 3$.
Explain what $f'(x) = 2x$ tells us about the f - function.
We can find the value of the derivative for a specific value *x*. For example for $x = 4$
$f'(4) = 2 \cdot 4 = 8$
What does the value 8 tell us about?

Question 5

In upper secondary school, the more 'formal' definition of the derivative concept was introduced.
We repeat the definition here:

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}.$$

Explain how you understand the definition. Then use the definition to show that $(x^2)' = 2x$.

Question 6

a) Explain how we can <u>use</u> the derivative concept within the area of mathematics.
b) A bike is equipped with speedometer that measures the speed $v(t)$ at the point t.
What are the physical meaning of the given expressions?

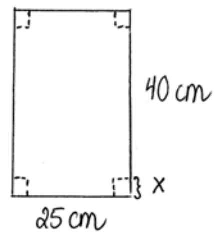
(1) $\dfrac{v(t_2)-v(t_1)}{t_2-t_1}$ and (2) $\lim\limits_{\Delta t\to 0}\dfrac{v(t+\Delta t)-v(t)}{\Delta t}$

Please, write (1) or (2) in the alternatives below:

- ☐ Average velocity
- ☐ Instantaneous velocity
- ☐ Average acceleration
- ☐ Instantaneous acceleration

Question 7

A rectangular plate has the sides of length 25 cm and 40 cm. We want to create a box by cutting away a square of every corner and folding up the side surfaces.
Call the square side x. See figure.

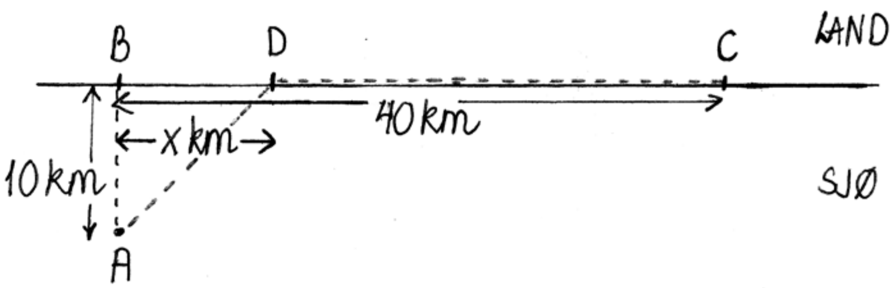


a) Show that the volume of the box can be expressed as
$V = 4x^3 - 130x^2 + 1000x$

b) Find the <u>largest</u> possible volume of the box.

Question 8

From oil platform in the point A a pipeline shall be laid to the receiving terminal for oil at point C. See the figure below.



The distance AB is 10 km and the distance BC is 40 km.

It is difficult to lay the pipeline in the sea and it is possible to add only 1 km per day there. On land it goes faster and it is possible to add 2 km line per day.
Assume that the pipeline runs from *A* via *D* to *C*. Call the distance *BD* for x.
Find a function *t(x)* that indicates the time required to lay the pipeline from *A* to *C*.
How can you determine the position of the point *D* in order to lay the pipeline as quick as possible?

Question 9

What tasks 1 – 8 were in your opinion difficult to answer? What tasks 1-8 were easy?
Please, justify your opinion.

Question 10

What is, in your opinion, the reason to have mathematics as subject in engineering education?
Please, choose two reasons from these stated below:

- ☐ Mathematics is interesting.
- ☐ Mathematics can be used in other subjects.
- ☐ One gets a better understanding of mathematics role in everyday/ work life.
- ☐ Mathematics develops logical thinking.
- ☐ It is important to get better at calculating.
- ☐ Other reasons

Explain____

____

Question 11

How do you perceive mathematics as a subject?

- ☐ Very difficult
- ☐ Difficult
- ☐ Sometimes hard/sometimes easy
- ☐ Easy
- ☐ Very easy
- ☐ No opinion

Comments ____

Question 12

What is, in your opinion, most important when you learn mathematics?

- ☐ Understanding
- ☐ Interest in the subject
- ☐ To see how math is used; application
- ☐ To get correct answers on the tasks
- ☐ Formulas and methods
- ☐ To learn to express mathematically the text problems and then solve them
- ☐ Other reasons

Explain ____

____

Question 13

Which of the following ways of working do you think works best for you while working with mathematics:

- ☐ Follow the lectures
- ☐ Work by your own
- ☐ Group work
- ☐ Other

____

____

Question 14

What do you think are the most important learning sources for you in the learning process?

Choose two main sources:

- ☐ Textbook
- ☐ Lecture notes

- ☐ Help from the teacher
- ☐ Cooperation/discussion with fellow students
- ☐ Other

____

____

Question 15

What expectations do you have for the calculus course?

- ☐ To pass exam
- ☐ To pass exam with good results
- ☐ To get better understanding of application of mathematics
- ☐ To be better in using algorithms and methods
- ☐ To get more interest in mathematics
- ☐ Other

____

____

Thank you very much for your answers!

Appendix C

Questions

1. What is your vision about the calculus textbook?
2. How do you decide the content of the book? What criteria do you use?
3. What is in your opinion, most important to consider when the new mathematical concept is introduced in the book?
4. What are, in your opinion, criteria for “a good definition”?
5. When shall a formal definition be introduced?
6. What preparations are necessary before introduction of formal definition?
7. What, in your opinion, develops mathematical intuition?
8. What kind of knowledge, conceptual or procedural, does your book emphasise?
9. When you write text for the book, who do you see as your prospective reader?
10. When you write, you use a special style of language. What do you want to achieve when you choose this style?
11. Could you imagine using other styles of languages and in such case which?
12. If you could imagine an ideal student using your book, how would the student act?
13. How does this ideal student differ from an average real student, according to your experience?
14. Who gives most responses to you as the author about the book
    - Students
    - Lectures
    - Other mathematicians?
15. What kind of changes did your do in the last edition of your book?
16. What were the reasons to make these changes?

Thank you very much for your answers.